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Measurement of Chladni mode shapes with optical lever method

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TITLE:
Measurement of Chladni Mode Shapes with an Optical Lever Method

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KEYWORDS:
Chladni pattern, vibration test, mode shape, optical lever method, circular plate, two-dimensional standing wave.

SUMMARY:
A simple method of measuring the Chladni mode shape on an elastic plate by the principle of an optical lever is proposed.

ABSTRACT:
Quantitatively determining the Chladni pattern of an elastic plate is of great interest in both physical science and engineering applications. In this paper, a method of measuring mode shapes of a vibrating plate based on an optical lever method is proposed. Three circular acrylic plates were employed in the measurement under different center harmonic excitations. Different from a traditional method, only an ordinary laser pen and a light screen made of

ground glass are employed in this novel approach. The approach is as follows: the laser pen projects a beam to the vibrating plate perpendicularly, and then the beam is reflected to the light screen in the distance, on which a line segment made of the reflected spot is formed. Due to the principle of vision persistence, the light spot could be read as a bright straight line. The relationship between the slope of the mode shape, length of the light spot and the distance of the vibrating plate and the light screen can be obtained with algebraic operations. Then the mode shape can be determined by integrating the slope distribution with suitable boundary conditions. The full-field mode shapes of Chladni plate could also be determined further in such a simple way.

INTRODUCTION:

Chladni mode shapes are of great interest in both science and engineering applications. Chladni patterns are reactions of physical waves, and one can illustrate the wave pattern with various methods. It is a well-known method to show the various modes of vibration on an elastic plate by outlining the nodal lines. Small particles are always employed to show the Chladni patterns, since they can stop at the nodes where the relative vibrating amplitude of the plate is zero, and the positions of the nodes vary with resonant mode to form various Chladni patterns.

Many researchers have paid attention to various Chladni patterns, but they only show the nodal lines of the mode shapes, the mode shapes (i.e., vibration amplitude) between the nodal lines are not illustrated. Waller investigated the free vibrations of a circle[1], a square[2], an isosceles right angled triangles[3], a rectangular[4], elliptical[5] plates, and different Chladni patterns are illustrated therein. Tuan et al. reconstructed different Chladni patterns through both experimental and theoretical approaches, and the inhomogeneous Helmholtz equation is adopted during the theoretical modeling[6,7]. It is a popular method of using Laser Doppler Vibrometer (LDV) or Electronic Speckle Pattern Interferometry (ESPI) to quantitatively measure the mode shapes of the Chladni patterns[8-10]. Although LDV enables femtometer amplitude resolution and very high frequency ranges, unfortunately, the price of LDV is also a little expensive for classroom demonstration and/or college physics education. With this consideration, the present paper proposed a simple approach to quantitatively determine the mode shapes of a Chladni pattern with low cost, since only an extra laser pen and a light screen are needed here.

The present measurement method is illustrated in **Figure 1**[11]. The vibrating plate has three different positions: the rest position, position 1 and position 2. Position 1 and 2 represent the two maximum vibrating places of the plate. A laser pen projects a straight beam on the surface of the plate, and if the plate locates at the rest position, the laser beam will be directly reflected to the light screen. While the plate locates at position 1 and 2, then the laser beam will be reflected to point A and B on the light screen, respectively. Due to the effect of persistence of vision, there will be a bright straight line on the light screen. The length of the bright light L is related to the distance D between the light screen and the location of the laser point. Different points on the plate have different slopes, which could be determined by the relationship between L and D. After obtaining the slope of the mode shape at different points on the plate, the problem turns into a definite integral. With the help of the boundary vibration amplitude of the plate and the discrete slope data, the mode shape of the vibrating plate can be obtained easily. The whole experimental setup is given in **Figure 2**[11].

This paper describes the experimental setup and procedure for the optical lever method to measure the Chladni mode shapes. Some typical experimental results are also illustrated.

PROTOCOL:

1. Experimental setup and procedures

NOTE: Set up the experimental system as shown in **Figure 2**.

1.1. Preparation of the vibration system

1.1.1. Prepare three 1.0-mm-thickness mirrored circular acrylic plates with diameter of 150 mm, 200 mm and 250 mm respectively. Drill a hole of 3 mm in diameter at the center of each plate. Mark several black points every 5 mm along an arbitrary radius.

1.1.2. Attach each plate to the actuate bar of the vibrator with a bolt in the middle point. Drive the vibrator with a sine wave using a waveform generator, and default settings will be enough for the resonance experiment.

NOTE: The excitation direction of the vibrator is horizontal for the convenience of moving the screen afterwards.

1.1.3. Acquisition of the resonance frequency

1.1.3.1. Place the laser pen to project the laser beam to the vibrating plate perpendicularly such that the beam is reflected to the light screen in the distance. The distances between the laser pen and the plate and the light screen are 120 mm and 500 mm, respectively.

NOTE: The farther the distance between the light screen and the vibrating plate, the more obvious the phenomenon appears. It is also noted that the present method can be used to measure either axisymmetric or non-axisymmetric mode shapes. Due to the consideration of simplicity and convenience, the present manuscript only demonstrates the application in determining axisymmetric mode shapes of three circular plates. Then we just need to measure the vibration amplitude along any radial direction to reconstruct the two-dimensional mode shape of the plate.

1.1.3.2. Move the laser pen along the direction perpendicular to its length direction to make the incident point scan over a diameter while the signal generator changing its frequency continuously. Do it quickly until the spot length is significantly stretched along the diameter when scanning in a certain frequency range, and some spots with almost no expansion appear. For the plate with a diameter of 150 mm, 200 mm and 250 mm, the frequency ranges swept are

200-400 Hz, 100-300 Hz and 50-250 Hz, respectively.

1.1.3.3. Scan this certain frequency range slowly and pick out the frequency at which the spot expands most obviously. It is found that for the plate with a diameter of 150 mm, 200 mm and 250 mm, the resonance frequencies are 346 Hz, 214 Hz and 150 Hz, respectively.

1.2. Preparation of the light path and measurement system

1.2.1. Place the light screen parallel to the vibrating plate. Mark the distance with a meter ruler, and use 500 mm as the starting distance.

1.2.2. Place the laser pen to project the beam perpendicularly on the plate such that the beam is reflected to the light screen in the distance. Make sure that the mark made before can be scanned while the laser pen is moving.

NOTE: The laser beam light must be projected perpendicularly on the plate.

1.3. Experimental measurement

1.3.1. Turn on the signal generator and set the excitation frequency to be the same as the resonance frequency obtained in step 1.1.3.3. The signal intensity should as small as possible once the light spot on the light screen is large enough to be recorded.

1.3.2. Adjust the laser pen to make the incident point coincide with the first marker, which is the nearest marker to the fixed point of the plate.

1.3.3. Move the screen from a distance *D* of 500 mm to 1000 mm and measure the spot length *L* on the screen every 50 mm. Record data in tabular form.

1.3.4. Adjust the laser pen to make the incident point adjacent to the next marker in turn and repeat step 1.3.3 until all the markers have been measured.

NOTE: Since acrylic plates are easily deformed plastically under excitation, the experimental measurement process of one plate cannot be paused for a long time.

1.3.5. Replace the former plate with the next one and repeat steps 1.3.1 to 1.3.4.

2. Data processing

2.1. Determine the angle θ between the incident and reflected light with relationship:

$$L=(D-w)\tan\theta+(D+w)\tan\theta=2D\tan\theta\approx 2D\theta \quad (1)$$

where *D* is the distance between the rest position of the vibrating plate and the light screen, *w* is vibrating amplitude of the plate, and *L* is the length of the light spot on the light screen. Several pairs of *D* and *L* are obtained in step 1.3.3.

2.2. Determine the slope $\frac{\mathrm{d}w}{\mathrm{d}r}$ of the mode shape by:

$$\frac{\mathrm{d}w}{\mathrm{d}r}=\frac{\theta}{2}=\frac{1}{4}\frac{L}{D} \tag{2}$$

NOTE: The obtained slope is always positive with Eqs.(1) and (2).

2.3. Use a minus sign between two zero points to obtain the true slope distribution.

NOTE: It does not matter whether the revision begins from the first or the second zero point.

2.4. Integrate the slope distribution of each plate and determine the integral constant by the nodes to obtain the mode shape with:

$$\frac{w_{n+1}-w_n}{r_{n+1}-r_n}=\left.\frac{\mathrm{d}w}{\mathrm{d}r}\right|_{r=r_{n+1}} \qquad r_0=0,\ \ w_0=w(0) \tag{3}$$

NOTE: Nodes correspond to the largest slope of the mode shape. $w(0)$ is a constant determined by the location of the nodal lines of the Chladni pattern shown in **Figure 2**.

2.5. Compute the uncertainty of the slope[12] with:

$$U_{w'}=\frac{1}{4}\sqrt{\left(t_{0.95}(n-2)\cdot s_{\mathrm{r}}\right)^2+\left(\frac{\sqrt{3}}{2}\frac{U_{\mathrm{m}}}{\left|D_i-\bar{D}\right|_{\max}}\right)^2} \tag{4}$$

NOTE: $t_{0.95}(n-2)$ is the *t* distribution factor with 95% confidence and degrees of freedom *n*-2, and it is about 2 here. s_{r} is the standard error of the linear regression with *D* and *L*, U_{m} denotes the uncertainty of the measured distance D_i, and is 0.5 mm here. The average measured distance is defined by $\bar{D}=\sum_{i=1}^{n}D_i$, and *n* denotes the total number of measured D_i.

REPRESENTATIVE RESULTS:

The excitation frequency that can excite axisymmetric Chladni pattern is determined through the frequency sweeping test. Three circular acrylic plates with diameters of 150 mm, 200 mm and 250 mm are tested, and results show that the first order axisymmetric resonance frequencies are 346 Hz, 214 Hz and 150 Hz for the three plates respectively. It is concluded that with larger diameter, the plate is more flexible, and the corresponding resonance frequency will be smaller. The Chladni patterns of the acrylic plate with different diameters are given in **Figure 3**[11].

Under the corresponding resonant frequency, the length of the light spot on the light screen of different plates can be measured and recorded. The regression value of mode shape slope can be obtained with Eq.(1), whose distributions along the radial direction of plate A, B and C are given in **Table 1**[11], and they are determined by measuring several different light spot lengths *L* of the specific laser point with different distance *D*.

Numerical simulation with ANSYS is performed to verify the present experimental results. The script code of APDL (ANSYS Parametric Design Language) is provided as a **Supplemental File 1**. **Figure 4**[11] shows the comparisons of the present experimental results and numerical results on the mode shape of different plates. It is very clear that all results with different conditions compare very well, which prove the feasibility of the present method in measuring the mode shape of plates.

FIGURE AND TABLE LEGENDS:

Figure 1: Illustration of the present measurement method. The basic measurement principal is illustrated in this figure, with an emphasis on the incident and reflect light beam and the relationship of different geometric parameters.

Figure 2: The experimental setup. The picture of experimental setup is provided for clearly understand and replicate the measurement approach easily.

Figure 3: Chladni pattern of different acrylic plates: (a) 150 mm, (b) 200 mm, (c) 250 mm. The Chladni patterns of three different acrylic circular plates are given respectively. The brown particles are sands and clearly show the nodal line of the Chladni patterns.

Figure 4: Comparisons of experimental results and numerical simulation for the mode shapes of different plates: (a) 150 mm, (b) 200 mm, (c) 250 mm. The numerical results obtained with ANSYS and the present experimental results are compared to verify the reliability of the present experimental method.

Table 1: Slope distribution of the mode shape along radial direction. The calculated slope distribution of the mode shape along the radial direction is provided, and both original and revised slope are given to illustrate the process of revision.

Supplement File 1: ANSYS script for simulating the dynamic response and mode shape of a plate.

DISCUSSION:

The optical lever method is adopted in this paper to determine the mode shape of a plate, since the Chladni pattern can only show the nodal lines of a vibrating plate. To determine the mode shape of the plate, the relationship between the slope and distance of the light screen and spot length should be obtained in advance. Then through definite integration calculation, the mode shape of the Chladni pattern could be quantitatively determined.

Generally, the whole process of the present approach includes the following steps: (1) Perform the forced vibration test to obtain the resonance frequency of the plate. (2) Conduct forced vibration test near the resonance frequency, and record the coordinates of the nodes of Chladni pattern. These data are used for calibrating the absolute mode shape obtained by experimental tests. (3) The laser spot is perpendicularly projected to different radial locations of the plate, and the length of the light spot on the light screen is measured. This test needs to be repeated several times with different distances between the vibrating plate and light screen to obtain the linear regression value of the mode shape slope with Eq. (2). (4) Obtain the experimental mode shape of the Chladni pattern with Eq. (4) through post processing the raw experimental data.

It should be pointed out that, although the present experimental demonstration only shows the measurement of axisymmetric Chladni patterns, it also could be used for the determination of nonaxisymmetric Chladni patterns in a forward manner. Not only circular plates, but also other shapes, such as triangle, rectangular, and even irregular shapes could be employed to show the beauty of Chladni patterns. Furthermore, if the measuring point density, laser source, measuring tool, as well as the integral calculation method are carefully chosen, the accuracy of the proposed method could be adapted to required level.

ACKNOWLEDGMENTS:

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DISCLOSURES:

The authors have nothing to disclose.

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Figure 1 Click here to access/download;Figure;Fig.1.eps

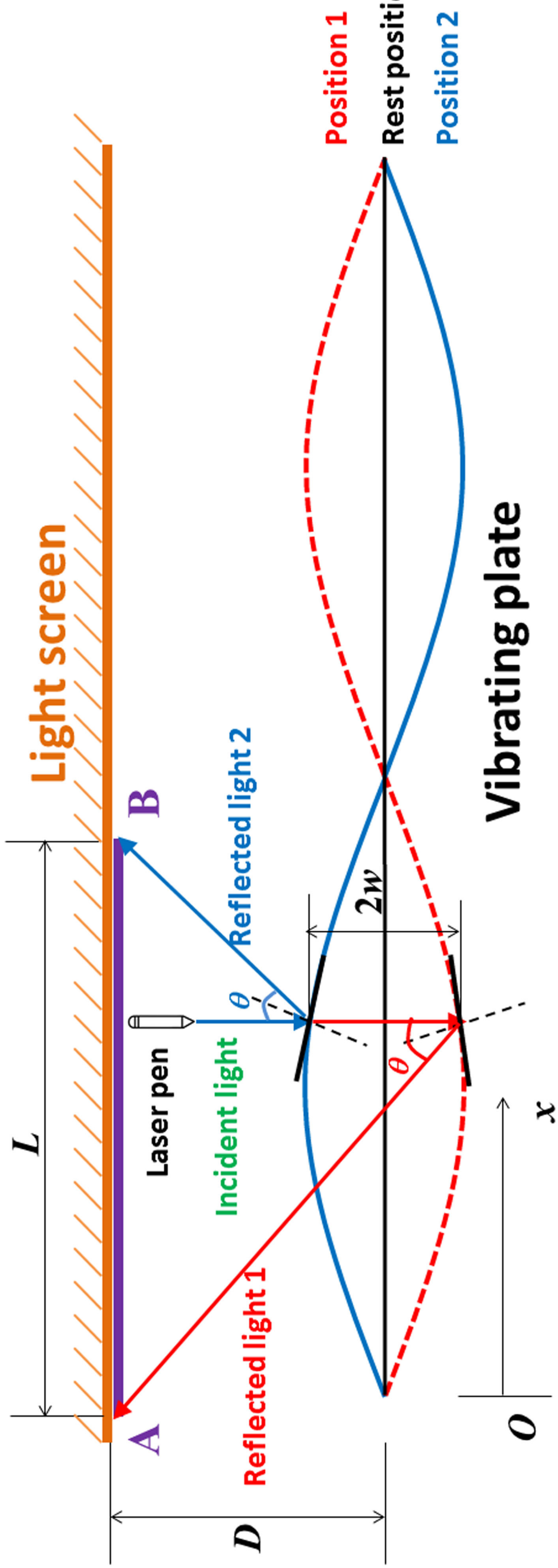


Figure 2

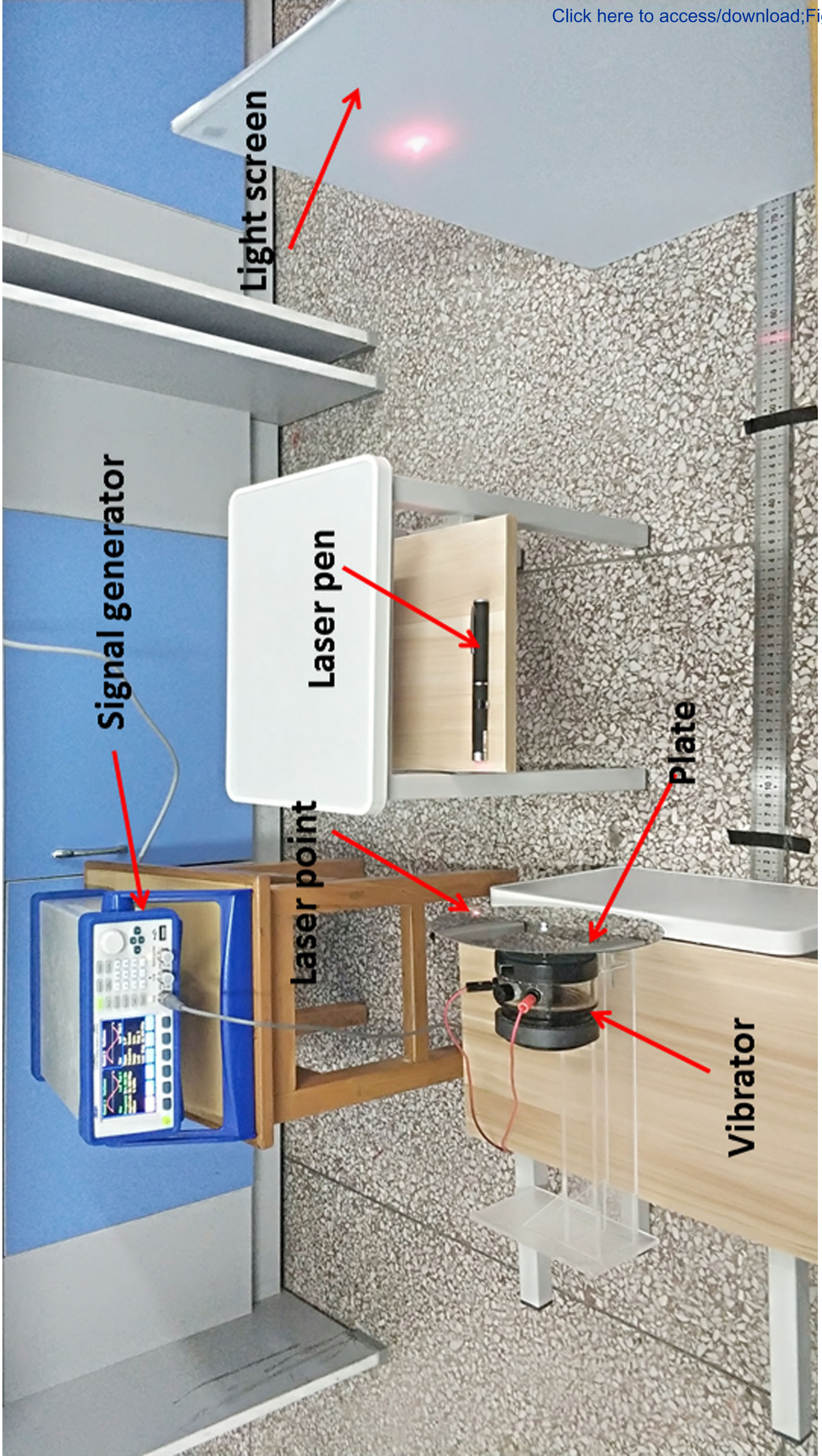
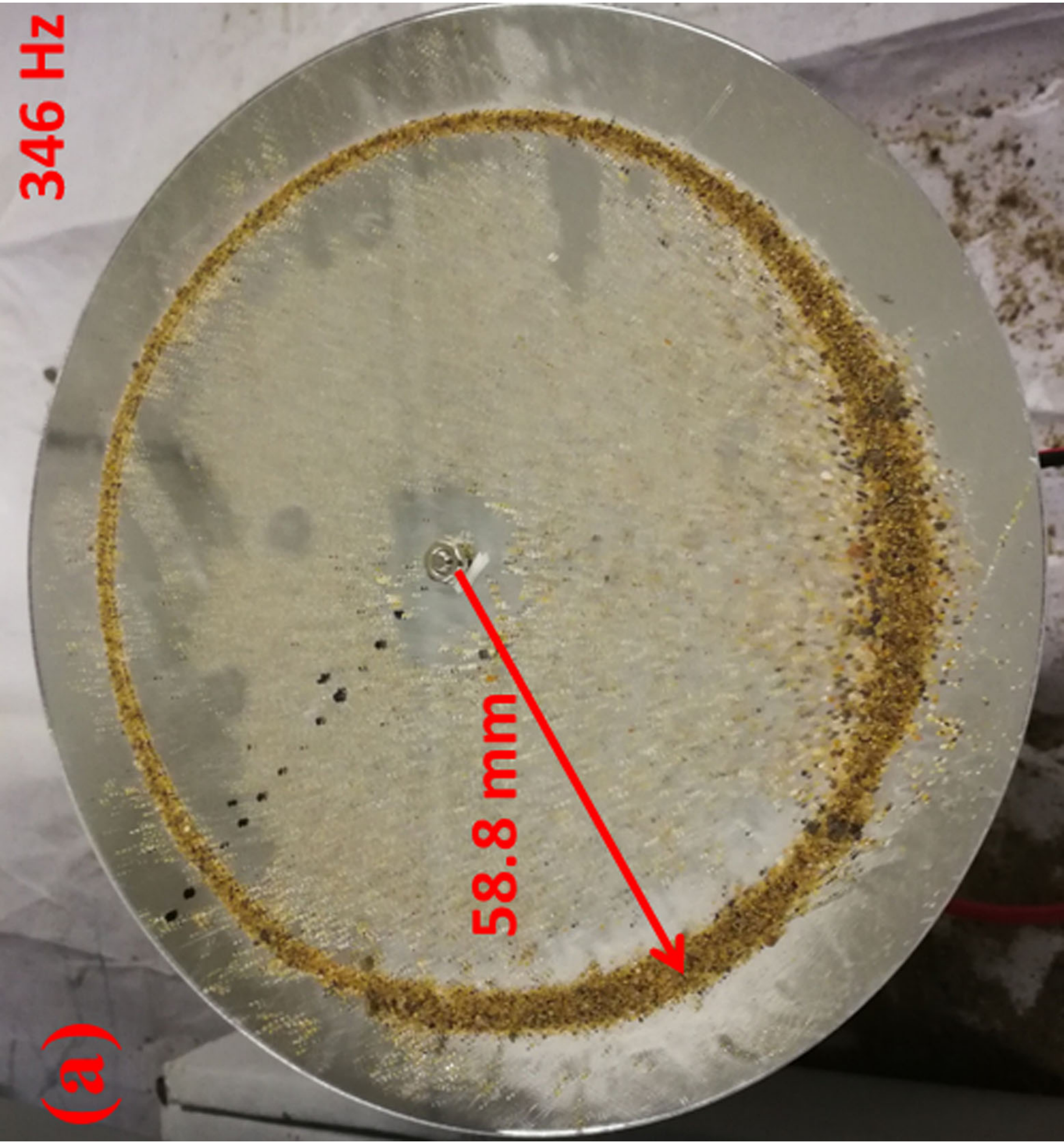

Light screen
Signal generator
Laser pen
Laser point
Plate
Vibrator

Figure 3a

Click here to access/download;Figure;Fig.3(a).eps



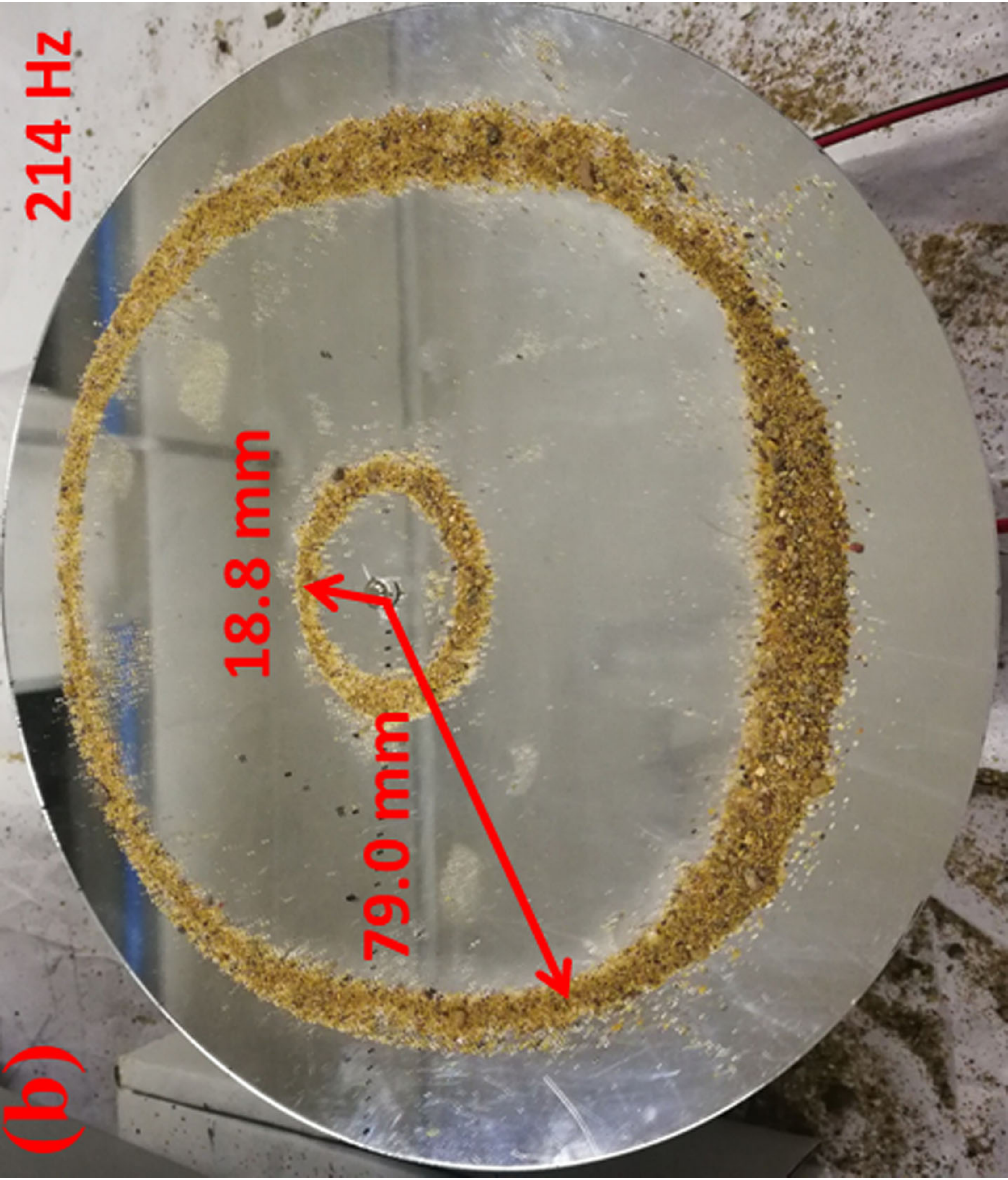
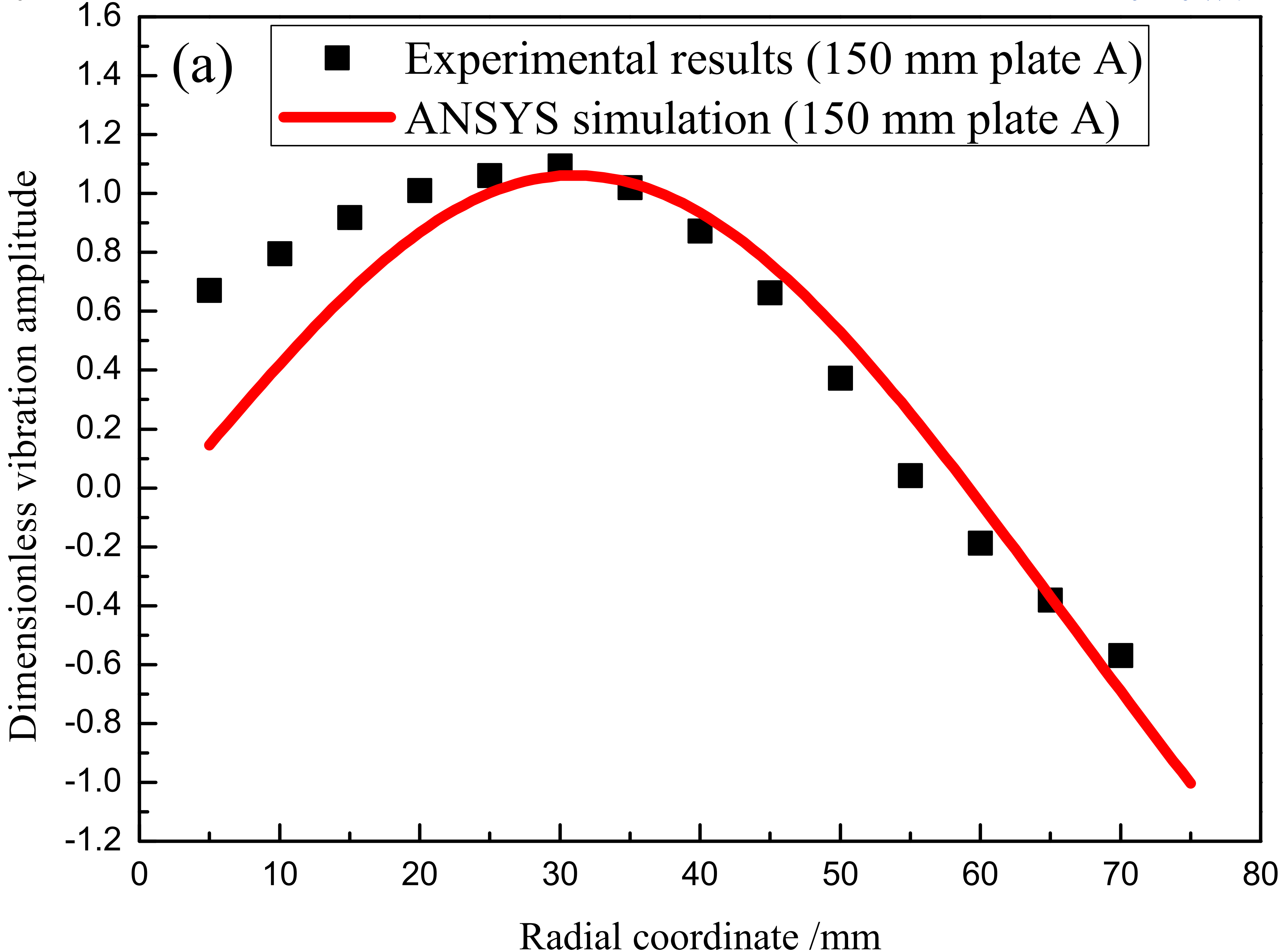

214 Hz
18.8 mm
79.0 mm
(b)

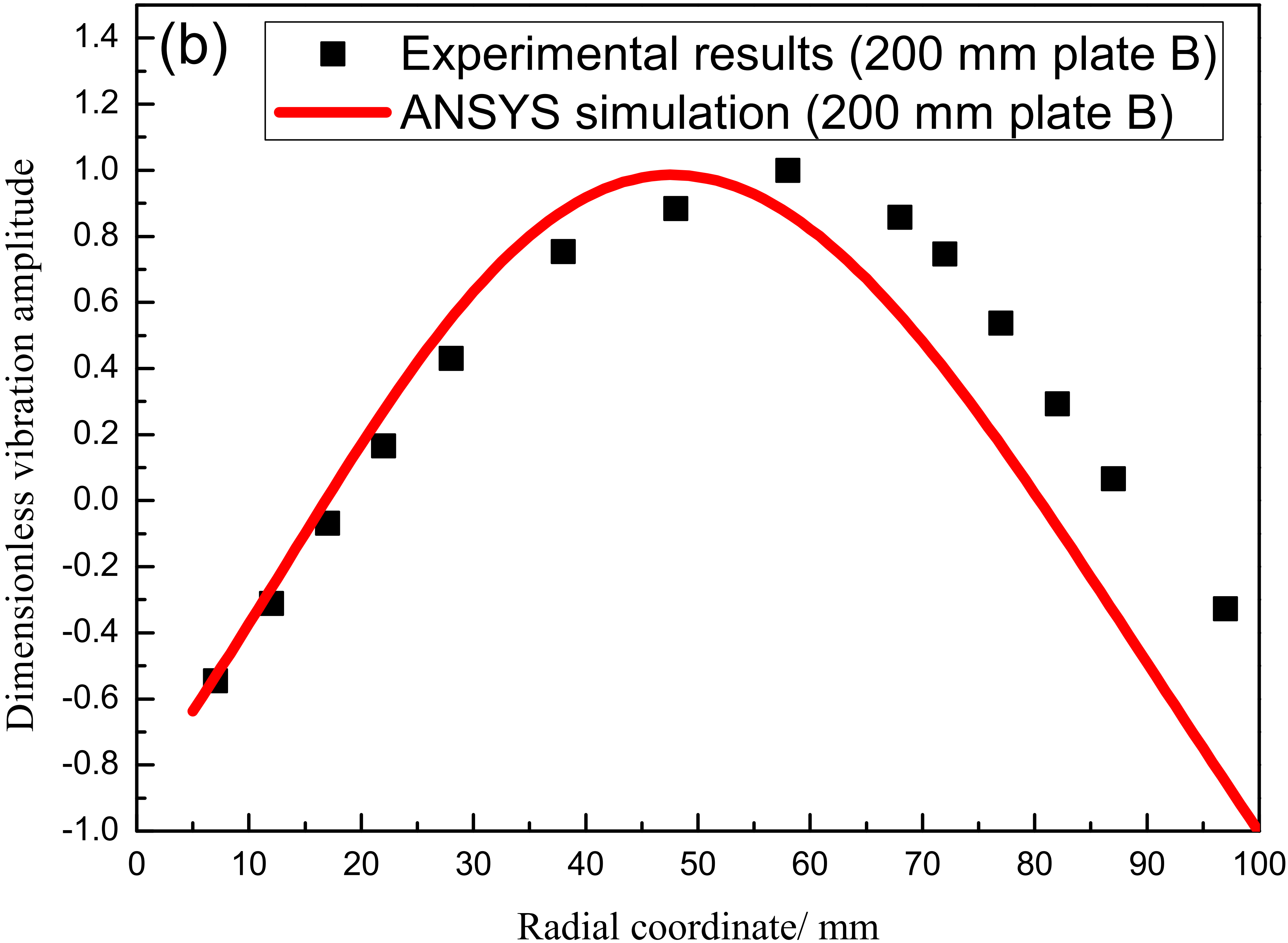
Figure 3c

Click here to access/download;Figure;Fig.3(c).eps

Figure 4a



(a)
Experimental results (150 mm plate A)
ANSYS simulation (150 mm plate A)
Dimensionless vibration amplitude
Radial coordinate /mm
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
0
10
20
30
40
50
60
70
80


(b)
Experimental results (200 mm plate B)
ANSYS simulation (200 mm plate B)
Dimensionless vibration amplitude
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0
10
20
30
40
50
60
70
80
90
100
Radial coordinate/ mm

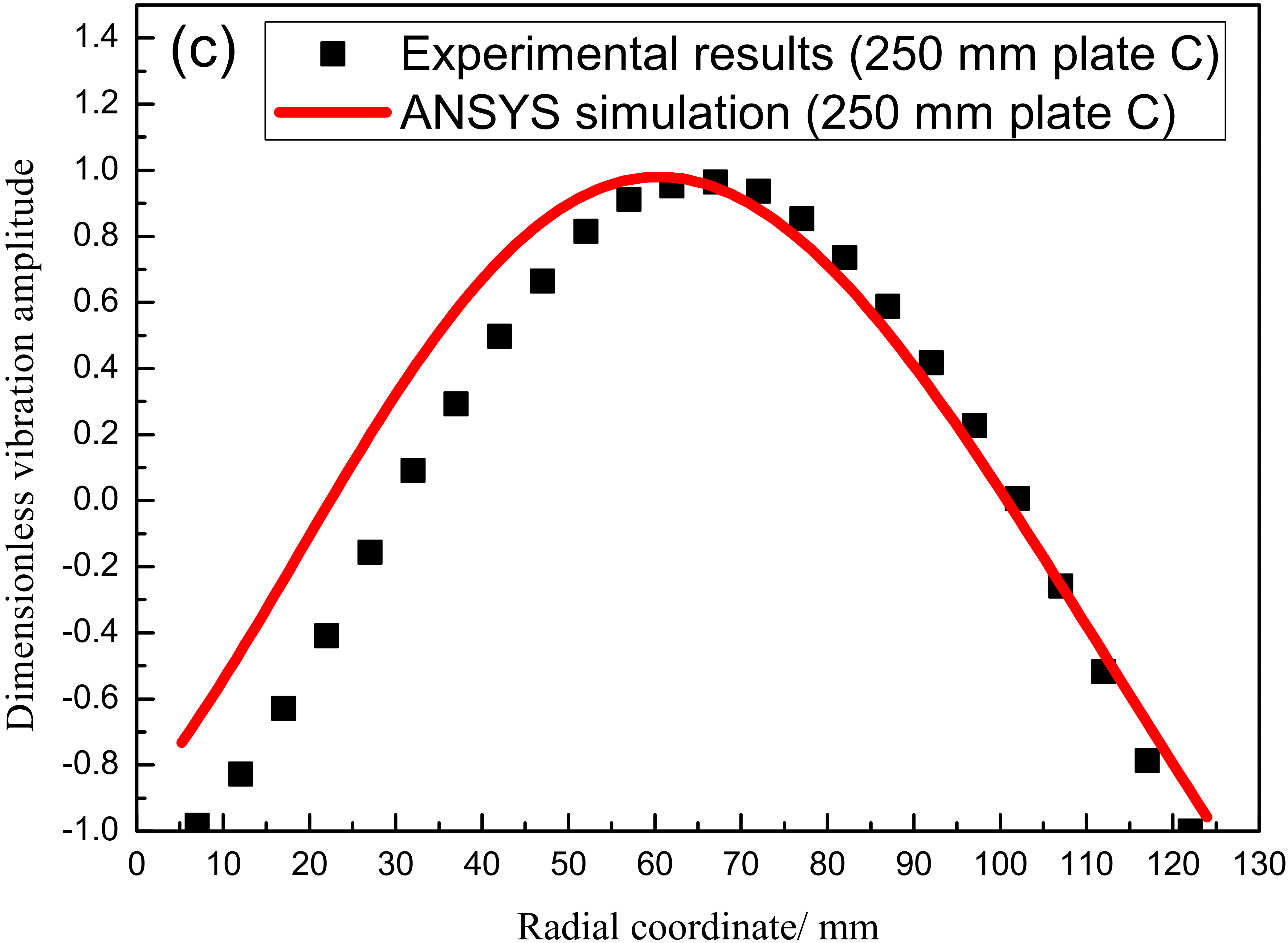

(c)
Experimental results (250 mm plate C)
ANSYS simulation (250 mm plate C)
Dimensionless vibration amplitude
Radial coordinate/ mm
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0
10
20
30
40
50
60
70
80
90
100
110
120
130

Table 1 Click here to access/download;Table;Table 1.xlsx

Plate A (Diameter=150 mm)			Plate B (Diameter=200 mm)			Plate C (Diameter=250 mm)		
r /mm	Directly calculated slope	Revised slope	*r* /mm	Directly calculated slope	Revised slope	*r* /mm	Directly calculated slope	Revised slope
5	0.00191	0.00191	7	0.00267	0.00267	7	0.0013	0.0013
10	0.00148	0.00148	12	0.00269	0.00269	12	0.00161	0.00161
15	0.00144	0.00144	17	0.00279	0.02785	17	0.00206	0.00206
20	0.00109	0.00109	22	0.00269	0.00269	22	0.00228	0.00228
25	0.00061	0.00061	28	0.00254	0.00254	27	0.00262	0.00262
30	0.00039	0.00039	38	0.00186	0.00186	32	0.00256	0.00256
35	0.00088	-0.00088	48	0.00075	0.00075	37	0.00209	0.00209
40	0.00173	-0.00173	58	0.00067	0.00067	42	0.00213	0.00213
45	0.00248	-0.00248	68	0.00082	-0.00082	47	0.00172	0.00172
50	0.00343	-0.00343	72	0.00158	-0.00158	52	0.00157	0.00157
55	0.00389	-0.00389	77	0.00241	-0.00241	57	0.001	0.001
60	0.00271	-0.00271	82	0.00281	-0.00281	62	0.00418	0.00418
65	0.00228	-0.00228	87	0.0026	-0.0026	67	0.00118	0.00118
70	0.00222	-0.00222	97	0.00226	-0.00226	72	0.00283	-0.00283
						77	0.00087	-0.00087
						82	0.00121	-0.00121
						87	0.00154	-0.00154
						92	0.00176	-0.00176
						97	0.00198	-0.00198
						102	0.00228	-0.00228
						107	0.00275	-0.00275
						112	0.00269	-0.00269
						117	0.00278	-0.00278
						122	0.00222	-0.00222

Name of Material/ Equipment	Company	Catalog Number
Acrylic plates	Dongguan Jinzhu Lens Products Factory	ıree 1.0-mm-thickne
Laser pen	Deli Group	2802
Light screen	Northern Tempered Glass Custom Taobao Store	
Ruler	Deli Group	DL8015
Signal generator	Dayang Science Education Taobao Store	TFG6920A
Vibrator	Dayang Science Education Taobao Store	‹imum amplitude is

Comments/Description

ess mirrored circular acrylic plates with diameter of 150 mm, 200 mm and 250 mm respectively. They are easily deformed.

Red laser is more friendly to the viewer. The finer the laser beam, the better.

Several layers of frosted stickers can be placed on the glass to achieve the effect of frosted glass.

The length is 1m and the division value is 1mm.

Common ones in university laboratories are available.

1.5cm.The power is large enough to cause a noticeable phenomenon when the board vibrates. Otherwise, add a power amplifier.

Following are our responses about reviewers' comments to our manuscript JoVE-61020-R1. We are grateful to the editor and reviewers for their constructive comments and suggestions on the revision of the manuscript. We have made all the necessary changes as suggested by the reviewers. All the revisions in the manuscript have been highlighted in red color.

Editor:

Comment 1: Please take this opportunity to thoroughly proofread the manuscript to ensure that there are no spelling or grammar issues. The JoVE editor will not copy-edit your manuscript and any errors in the submitted revision may be present in the published version.

Response: Thanks for the reminder. The authors carefully proofread the manuscript and do our best to ensure the quality of the language.

Comment 2: Please revise lines 58-60, 66-67, 72-73, 85-87, 202-203, 235-237, 244-248 to avoid textual overlap with previously published work.

Response: According to the editor's suggestion, the following lines are revised to avoid textual overlap with previously published work.

Location	Original text	Revised text
L58-60	The small particles (such as sand, salt, etc.,) can stop at the nodes of the resonant modes on a vibrating plate, and different resonant mode forms different Chladni patterns.	Small particles are always employed to show the Chladni patterns, since they can stop at the nodes where the relative vibrating amplitude of the plate is zero, and the positions of the nodes vary with resonant mode to form various Chladni patterns.
L66-67	Based on the inhomogeneous Helmholtz equation, Tuan et al. successfully reconstructed the experimental Chladni patterns6, 7,	Tuan et al. reconstructed different Chladni patterns through both experimental and theoretical approaches, and the inhomogeneous Helmholtz equation is adopted during the theoretical modeling [6, 7].
L72-73	the present paper proposed a rather simple and low-cost approach to quantitatively determine the mode	the present paper proposed a simple approach to quantitatively determine the mode shapes of a Chladni pattern

	shapes of a Chladni pattern with optical lever method, and only an extra laser pen and a light screen are needed here.	with rather low cost, since only an extra laser pen and a light screen are needed here.
L85-87	After obtaining the slope of the mode shape at different point of the plate, then the mode shape could be calculated by integrating the slope easily with a chosen boundary value.	After obtaining the slope of the mode shape at different points on the plate, the problem turns into a definite integral. With the help of boundary vibration amplitude of the plate and the discrete slope data, the mode shape of the vibrating plate can be obtained easily.
L202-203	The comparisons of the present experimental results and numerical results are given in Figure 4[10]. It is very clear that the experimental results and the numerical simulation compare very well	**Figure 4**[11] shows the comparisons of the present experimental results and numerical results on the mode shape of different plates. It is very clear that all results with different conditions compare very well,
L235-237	Firstly, the relationship between the slope of the mode shape and distance of the light screen is obtained. Next, by integrating the slope data with suitable boundary conditions,	To determine the mode shape of the plate, the relationship between the slope and distance of the light screen and spot length should be obtained in advance. Then through definite integration calculation,
L244-248	(3) Obtain the linear regression value of the mode shape slope with Eq.(2) through the measurements of the length of the light spot perpendicularly projected on the light screen from different radial locations. This process should be repeated several times by changing the distances between the vibrating plate and light screen to obtain the linear regression value.	(3) The laser spot is perpendicularly projected to different radial locations of the plate, and the length of the light spot on the light screen is measured. this test needs to be repeated several times with different distances between the vibrating plate and light screen to obtain the linear regression value of the mode shape slope with Eq.(2).

Comment 3: Please add more details to your protocol steps. There should be enough detail in each step to supplement the actions seen in the video so that viewers can easily replicate the protocol. Please ensure you answer the "how" question, i.e., how is the step performed? Alternatively, add references to published material specifying how to perform the protocol action. Please be as specific as you can with respect to your experiment providing all necessary details. See examples below.

1.1.2: At what speed is the vibrator driving? Please specify the operating settings of the waveform generator.

Response: Thanks for the suggestion. For the purpose of finding resonance frequency of the plate, default settings of the waveform generator and vibrator would be enough and no special attention should be paid.

Lines 109-111:

1.1.2. Attach each plate to the actuate bar of the vibrator with a bolt in the middle point. Drive the vibrator with a sine wave using a waveform generator, and default settings will be enough for the resonance experiment.

1.1.3.1: Please specify the distance from the laser pen to the vibrating plate/light screen.

Response: The distances are given in the revised manuscript.

Lines 118-120:

1.1.3.1. Place the laser pen to project the laser beam to the vibrating plate perpendicularly and the beam is reflected to the light screen in the distance. The distances between the laser pen and the plate and the light screen are 120 mm and 500 mm respectively.

1.1.3.2: Moving the laser pen in which direction? Please specify the frequency range of the signal generator.

Response: The revised manuscript is as follows:

Lines 130-135:

1.1.3.2. Move the laser pen along the direction perpendicular to its length direction to make the incident point scan over a diameter while the signal generator changing its frequency continuously. Do it quickly until the spot length is significantly stretched along the diameter when scanning in a certain frequency range, and some spots with almost no expansion appear. For the plate with a diameter of 150 mm, 200 mm and 250 mm, the frequency ranges swept are 200-400 Hz, 100-300 Hz and 50-250 Hz respectively.

1.1.3.3: Please specify the frequency range and the final selected frequency.

Response: The frequency range is the same as obtained in step 1.1.3.2, and the final selected frequency is the resonance frequency of the plate. The revised manuscript is as follows:

Lines 137-139:

1.1.3.3. Scan this certain frequency range slowly and pick out the frequency at which the spot expands most obviously. It is found that for the plate with a diameter of 150 mm, 200 mm and 250 mm, the resonance frequencies are 346 Hz, 214 Hz and 150 Hz respectively.

1.2.1: What is the distance?

Response: The distance changes from time to time during the experiments and the starting distance is 500 mm. The revised manuscript is as follows:

Lines 143-144:

1.2.1. Place the light screen parallel to the vibrating plate. The distance is marked with a meter ruler, and 500 mm is the starting distance.

1.3.1: Please specify the frequency.

Response: The revised manuscript is as follows:

Lines 154-156:

1.3.1. Turn on the signal generator and set the excitation frequency to be the same as the resonance frequency obtained in step 1.1.3.3, the signal intensity should as small as possible once the light spot on the light screen is large enough to be recorded.

1.3.2: What is the first marker?

Response: The revised manuscript is as follows:

Lines 158-159:

1.3.2. Adjust the laser pen to make the incident point coincide with the first marker, which is the nearest marker to the fixed point of the plate.

2.1: Please define D, w in the equation. From which step are these obtained? It is unclear.

Response: The definitions of the parameters in Eq.(1) are given in the revised manuscript as follows:

Lines 177-179:

where D is the distance between the rest position of the vibrating plate and the light screen, w is vibrating amplitude of the plate, and L is the length of the light spot on the light screen. Several pairs of D and L are obtained in step 1.3.3.

Comment 12: Table of Materials: Please sort the materials alphabetically by material name.

Response: According to the editor’s suggestion, the materials had been sorted alphabetically by material name in Table of Materials.

Reviewer #1:

Comment 1: The manuscript reports an experimental technique for observing vibration patterns of circular plates (Chladni figures). The technique is based on measurements of a laser beam reflection angle from a vibrating surface in various points of the plate. To this end, the reflected optical field is projected on a distant screen and for consequent phases of the vibration stretches into a light strip. The length of the strip is evaluated and yields the value of a slope of the vibrating surface. An additional integration is required to recover the search vibration displacement pattern.

An overall impression on the manuscript is quite positive: unlike commercial laser vibrometry, it delivers a simple and low-cost illustrative technique that requires a minimal equipment to visualise and quantify vibration patterns.

Response: Thanks.

Comment 2: It is not clear how fast the technique is, i.e. how long it takes to calculate the pattern out of the experiment data for a single measurement point?

Response: The proposed method is very simple and fast, after obtaining the resonance frequency of the Chladni pattern to be measured, it only takes several minutes to record experiment data for a single measurement point. The post processing of the experimental data could be accomplished by excel software automatically, and it is really fast.

Comment 3: The same point about the sensitivity of the technique in terms of the vibration amplitude required for the shaker.

Response: Since the mode shape is independent with the vibration amplitude of the plate under linear assumption. During each test, the input signal of vibrator is tuned to ensure the vibration amplitude of the plate is small. In the other hand, on order to determine the vibration amplitude along the radial direction, we need to know the slope on each point in the radial direction, and this slope is determined by the relationship shown in Eq.(1). During the experiment, we measure several pairs of (*L*, *D*) in Eq.(2) for one data point, and then the slope could be determined by linear regression. Through this treatment, the accuracy of the slope is high.

$$\frac{\mathrm{d}w}{\mathrm{d}r} = \frac{\theta}{2} = \frac{1}{4}\frac{L}{D} \tag{2}$$

Comment 4: Fig. 3 b) and c) do not show the fundamental vibration mode of the plates while they are discussed in the text as if they were.

Response: Thanks, we are sorry for this negligence. In fact, the present method can be used to measure either axisymmetric or non-axisymmetric mode shapes. Due to the consideration of simplicity and convenience, the present manuscript only demonstrates the application in determining axisymmetric mode shapes. In the revised manuscript, we clarify this point.

Lines 122-128:

NOTE: The farther the distance between the light screen and the vibrating plate is, the more obvious the phenomenon appears. It is also noted that the present method can be used to measure either axisymmetric or non-axisymmetric mode shapes. Due to the consideration of simplicity and convenience, the present manuscript only demonstrates the application in determining axisymmetric mode shapes of three circular plates. Then we just need to measure the vibration amplitude along any radial direction to reconstruct the two-dimensional mode shape of the plate.

Lines 211-212:

and results show that the first order axisymmetric resonance frequencies are 346 Hz, 214 Hz and 150 Hz for the three plates respectively.

Comment 5: The *y*-axes in Fig. 4 must be explained: if this the displacement normalised to the amplitude in the center of the plate, then why does it become negative?

Response: As pointed out in 2.3, “2.3 A minus sign between two zero points is required to obtain the true slope distribution.”, the slope data is modified based on physical considerations, and some of the slopes turn from positive to negative. The displacement of the plate is obtained by integrating the slope, since slope could be positive or negative, then the normalized displacement or mode shape also could be positive or negative.

Comment 6: The reference to the original paper by Chladni must be referred to.

Response: The original paper on this subject is published in European Journal of Physics, and had already been cited as reference 10.

Comment 7: Some grammar errors must be traced and corrected: e.g. angel (p.3), can only shows (p.5)…

Response: Thanks for the suggestion, and the authors carefully read the manuscript to avoid any grammar errors and typos.

Reviewer #2:

Comment 1: This paper reports a rather simple and cost-effective setup for the measurement of mode shapes of vibrating membranes. The measurement procedure has been explained in details and in an understandable way. The measurement method is compared with FEM analysis of the vibrating membrane.

The setup is interesting, simple, and innovative. However, I see four major weaknesses in the paper: (1) the background of the work is not reviewed properly (for example see the comments 4 and 5). (2) The method is not verified by other existing measurement methods such as LDV or ESPI, but merely by FEM analysis. (3) The resolution of the method has not been discussed (for example see the comment 8). (4) The language of the paper should be improved substantially.

Response: Thanks for the comments and suggestions, the responses are given below in detail.

Comment 2: Line 55-56 is not correct. Chladni mode shapes are not the same as Chladni patterns. Line 56 is not correct. We can't say that acoustic waves are invisible.

Response: Thanks for the comment, we delete the following texts in the revised manuscript: ", and it is also called Chladni patterns" and word "invisible".

Comment 3: This sentence is unclear to me: "A lot of previously published papers had paid attentions on various Chladni patterns, but only the nodal lines of the Chladni patterns are illustrated, and the most valuable mode shapes are rarely considered in depth."

Response: This sentence is revised in the revised manuscript as follows:

Lines 64-66

Many researchers have paid attentions to various Chladni patterns, but they only show the nodal lines of the mode shapes, the mode shapes (i.e., vibration amplitude) between the nodal lines are not illustrated.

Comment 4: LDV can be used for microscale characterization of vibration amplitude, and this should be mentioned as an advantage of LDV in introduction.

Electronic Speckle Pattern Interferometry (ESPI) can be also used for vibration measurements similar to [1]. This can be mentioned in the paper.

[1] Georgas, P.J. and Schajer, G.S., 2013. Simultaneous Measurement of Plate Natural Frequencies and Vibration Mode Shapes Using ESPI. Experimental Mechanics, 53(8), pp.1461-1466.

Response: Thanks for the comment, the revised manuscript is as follows:

Lines 72-76

It is a popular method of using Laser Doppler Vibrometer (LDV) or Electronic Speckle Pattern Interferometry (ESPI) to quantitatively measure the mode shapes of the Chladni patterns[8-10]. Although LDV enables femtometer amplitude resolution and very high frequency ranges, unfortunately, the price of LDV is also a little expensive for classroom demonstration and/or college physics education.

Comment 5: The measurement system should be calibrated/compared with another experimental method such as LDV or ESPI, not with numerical simulation.

Response: The main intention of the present manuscript is to provide a simple method to quantitatively determine the mode shapes of vibrating plate. For isotropic thin plate, numerical results had been proved to have adequate accuracy on mode shape predictions. Meanwhile, the axisymmetric mode shape of circular plate is also well-known, and the present method is not meant to focus on accuracy of the detailed absolute value of the mode shape. So the experimental results are just compared with numerical results.

Comment 6: How did the authors select the amplitude of vibrating element (the external force amplitude) in the FEM models?

Response: In the FEM models, linear assumption is adopted during the numerical simulation. For modal analysis, no external force is needed; for harmonic response analysis, since linear assumption is adopted here, and the vibration amplitude of the plate is normalized with the maximum amplitude. The external force amplitude will not affect the normalized mode shape of the plate.

Comment 7: Provide mathematical calculation of the minimum measurable vibration amplitude by assuming the diameter of the laser beam, and compare it with alternative methods (LDV and ESPI).

Response: As we can see the uncertainty analysis in Eq.(4), several parameters could affect the accuracy of the slope of the mode shape, as well as the mode shape. Since all the tests are manually controlled, it is not easy to quantitatively determine the error. Although the diameter of the laser beam could affect the judgment of the total length of the light spot on the light screen, it is just one of the key effects that contribute to the accuracy of the measurement. Luckily, as we mentioned in response to comment 5, the main intention of the present manuscript is to provide a simple method to quantitatively determine the mode shapes of vibrating plate, and is not meant to focus on accuracy of the detailed absolute value of the mode shape. So results from the present method are not compared with results obtained by commercial instrument with LDV or ESPI.

Comment 8: The paper needs a major revision on English, and should be sent for proof-reading. I found many grammatical mistakes.

Response: The authors carefully proofread the manuscript and do our best to ensure the quality of the language. Thanks for the suggestion.

On behalf of all authors, I take the opportunity to express our appreciation to the reviewers and editor again for suggesting how to improve our manuscript.

Dear Donghuan Liu,

Thank you for your request to reproduce material published by IOP Publishing ***in "Measurement of Chladni mode shapes based on optical lever method."*** *to be published by* ***MYJoVE.***

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