Using Quantum Coherence for Remote Sensing

Marlan O. Scully

With A. Sokolov, P. Sprangle, and A. Svidzinsky

Texas A&M University   Princeton University

Paint Branch Distinguished Lecture in Applied Physics
University of Maryland, October 2015

RAMAN AT RANGE

1) Anthrax Detection via FAST CARS
    a) Motivation
    b) Coherent vs. Spontaneous Raman (Boyd, Shen, Welch)
2) Random Raman
    a) Remote Chemical Detection
    b) Bone Density Measurements

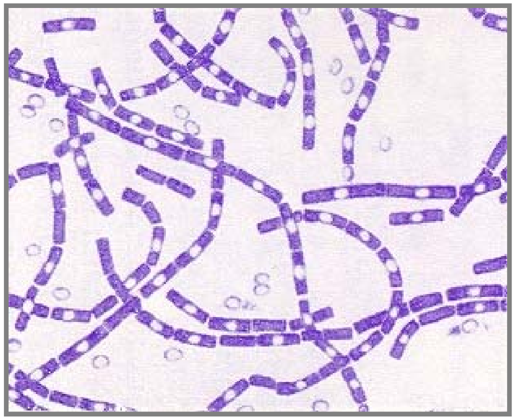
DICKE AT A DISTANCE

3) Superradiant Swept Gain
    a) High Gain Backward Lasing
    b) Coherence Brightened Air Laser
4) Backward Quantum Amplification by Superradiant Emission of Radiation
    a) Concept and Numerical Simulation
    b) Simple Gain Calculation
5) CARS in the Sky
6) Summary

NJ News Movie on Anthrax Detection

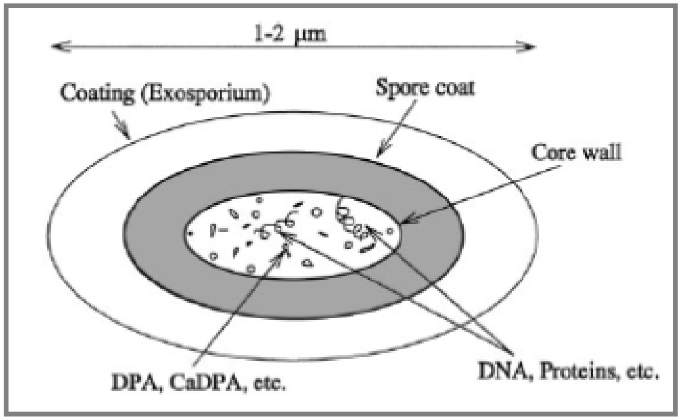
*M.O. Scully et al., PNAS **99**, 10994 (2002)*

FAST CARS: Engineering a laser spectroscopic technique for rapid identification of bacterial spores



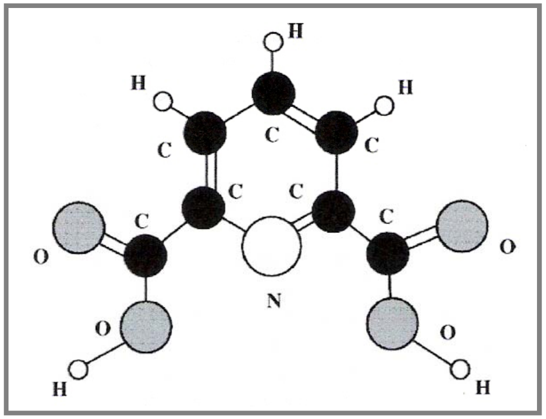
Bacillus anthracis

http://textbookofbacteriology.net



Sketch of a spore

M. O. Scully et al, PNAS, 2002



Dipicolinic acid

Marker-molecule!

FAST CARS – **F**emtosecond **A**daptive **S**pectroscopic **T**echnique for **C**oherent **A**nti-**S**tokes **R**aman **S**cattering

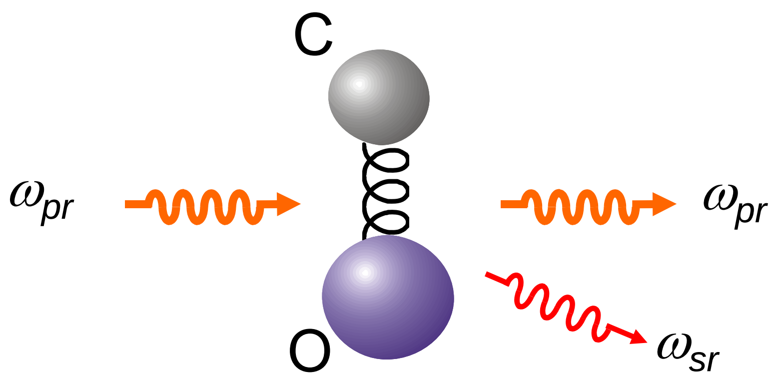
G. Beadie et al. (Washington, USA)

M. Mehendale et al. (Princeton, USA)

T. Siebert, W. Kiefer et al. (Wuerzburg, Germany)

Spontaneous Raman Scattering

Raman effect: experimentally discovered in February of 1928, by



all predicted theoretically by Adolf Smekal in 1923 !

Leonid Isaakovich Mandelshtam (1879-1944)

Grigory Samuilovich Landsberg

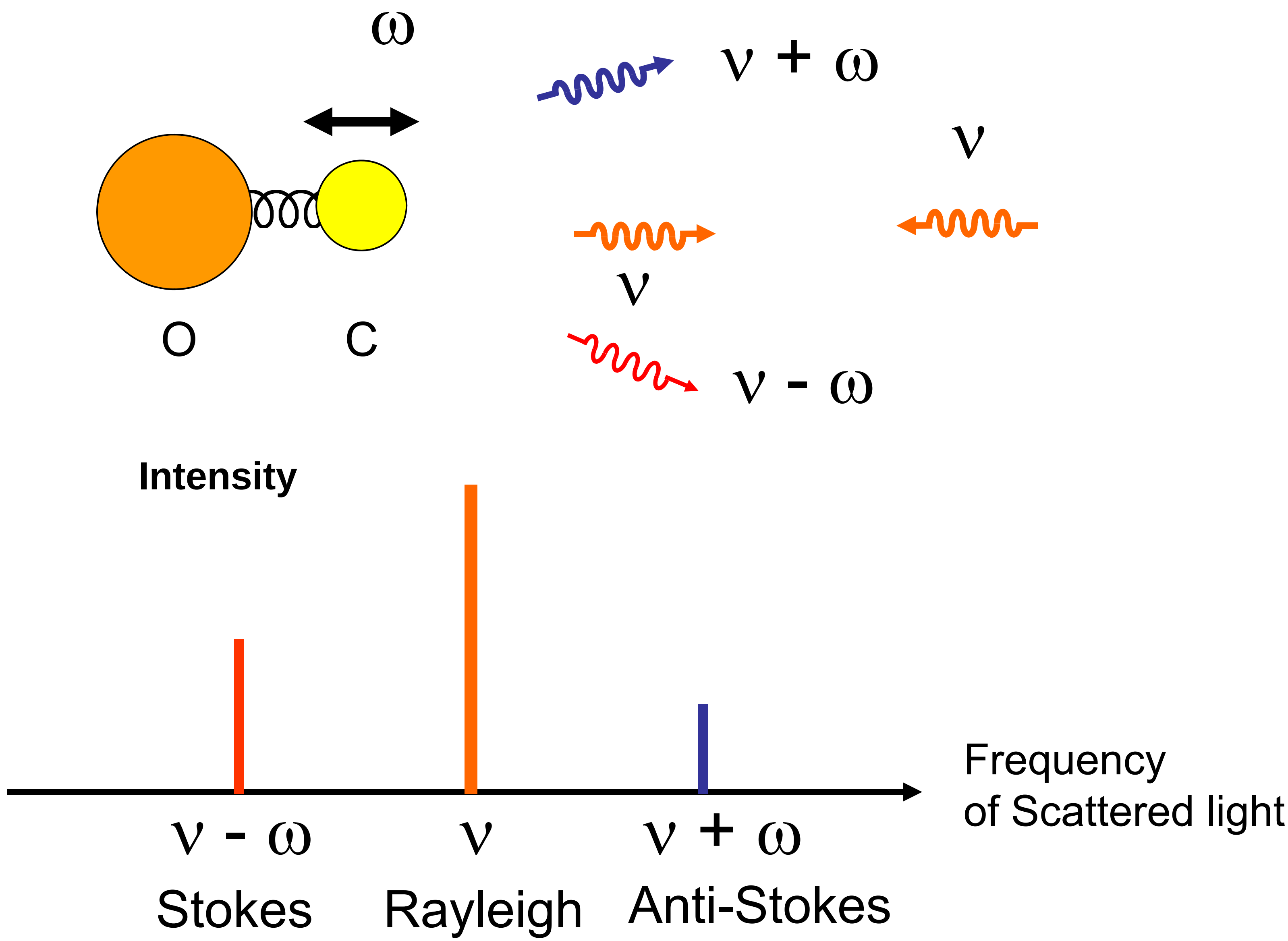
(1890-1957)

AND

Chandrasekhara Venkata Raman (1888-1970)

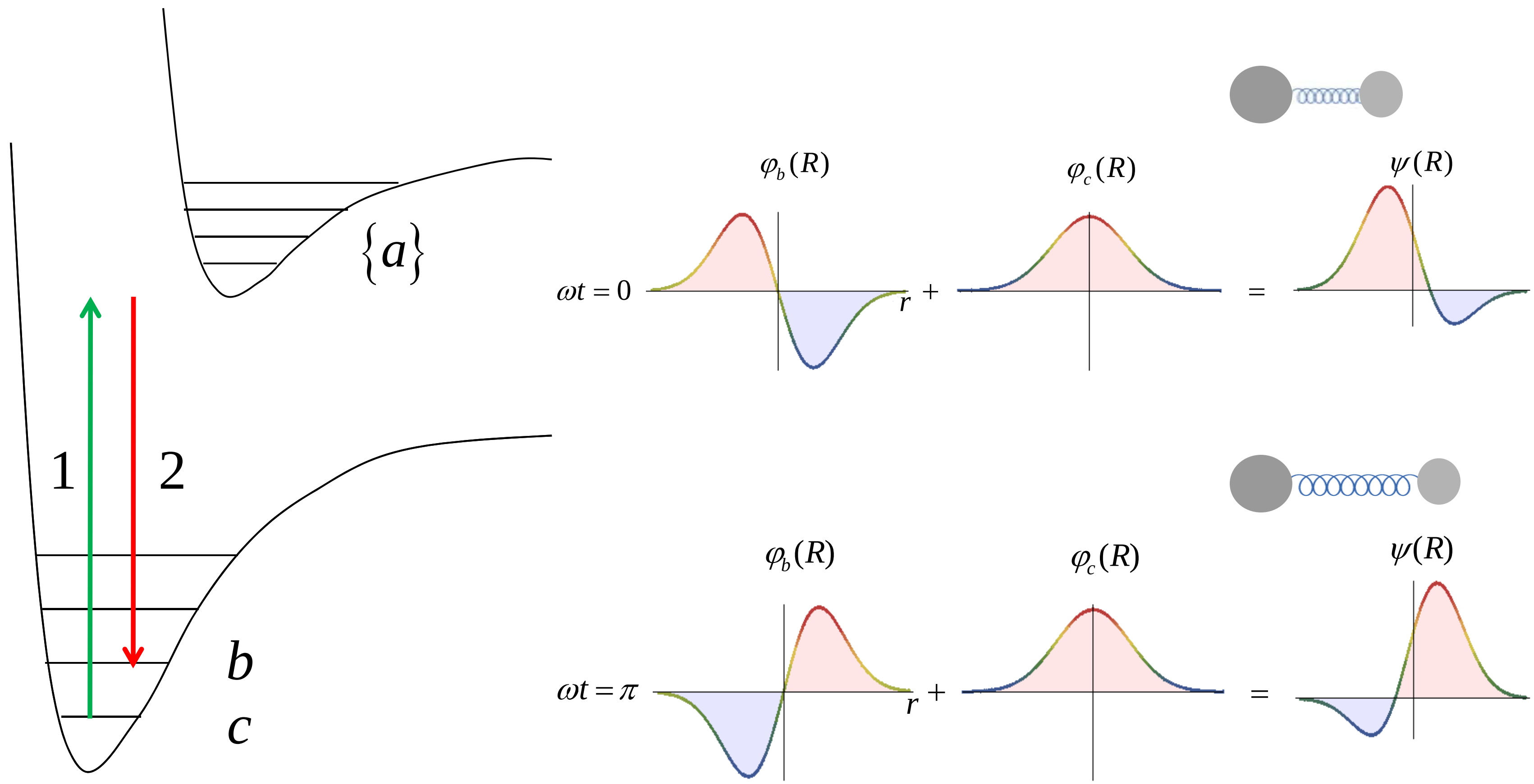
1930 Nobel in Physics

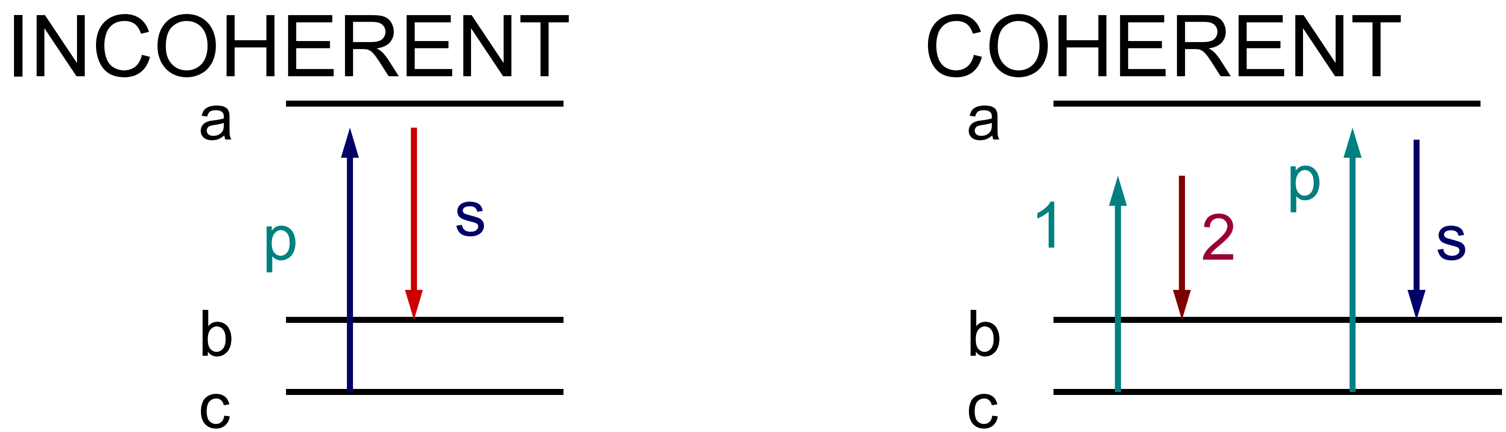
Raman Scattering is like Reflection off Oscillating Mirror



Raman Coherence, ρ_{bc}, implies Oscillation of Nuclear separation

$$\psi_{Nuc}(R,t) = Be^{-i\omega_b t}\varphi_b(R) + C\varphi_c(R)$$





Raman Hamiltonian

$$H_{p,s} = \sum_j G\left(\left|b\right\rangle\left\langle c\right|\right)_j a_s^{+} a_p e^{i\left(\vec{k}_p - \vec{k}_s\right)\cdot r_j} + adj.$$

Equation of motion for Stokes field

$$\dot{a}_s = -\frac{i}{\hbar}\left[H_{p,s}, a_s\right]$$

Short Pulse

$$a_s(\tau) \cong -\frac{i}{\hbar}\sum_j G\tau\left(\left|b\right\rangle\left\langle c\right|\right)_j a_s^{+} a_p e^{i\left(\vec{k}_p - \vec{k}_s\right)\cdot r_j}$$

Number of Stokes photons

$$\langle n_s(\tau)\rangle \cong \sum_{i,j}\left(\frac{G\tau}{\hbar}\right)^2 \left\langle \left(|c\rangle\langle b|\right)_i \left(|b\rangle\langle c|\right)_j \right\rangle \langle n_p\rangle e^{i(\vec{k}_p-\vec{k}_s)\cdot(\vec{r}_j-\vec{r}_i)}$$

$$\langle n_s(\tau)\rangle \cong \left\{\sum_{i}\rho_{cc}^{i} + \sum_{i\neq j}\rho_{bc}^{i}\rho_{cb}^{j} e^{i\left[\vec{k}_1+\vec{k}_p-\vec{k}_2-\vec{k}_s\right]\cdot(\vec{r}_j-\vec{r}_i)}\right\}\left(\frac{G\tau}{\hbar}\right)^2 \langle n_p\rangle$$

Spontaneous

Incoherent

One Atom

Cooperative

Coherent

N Atom

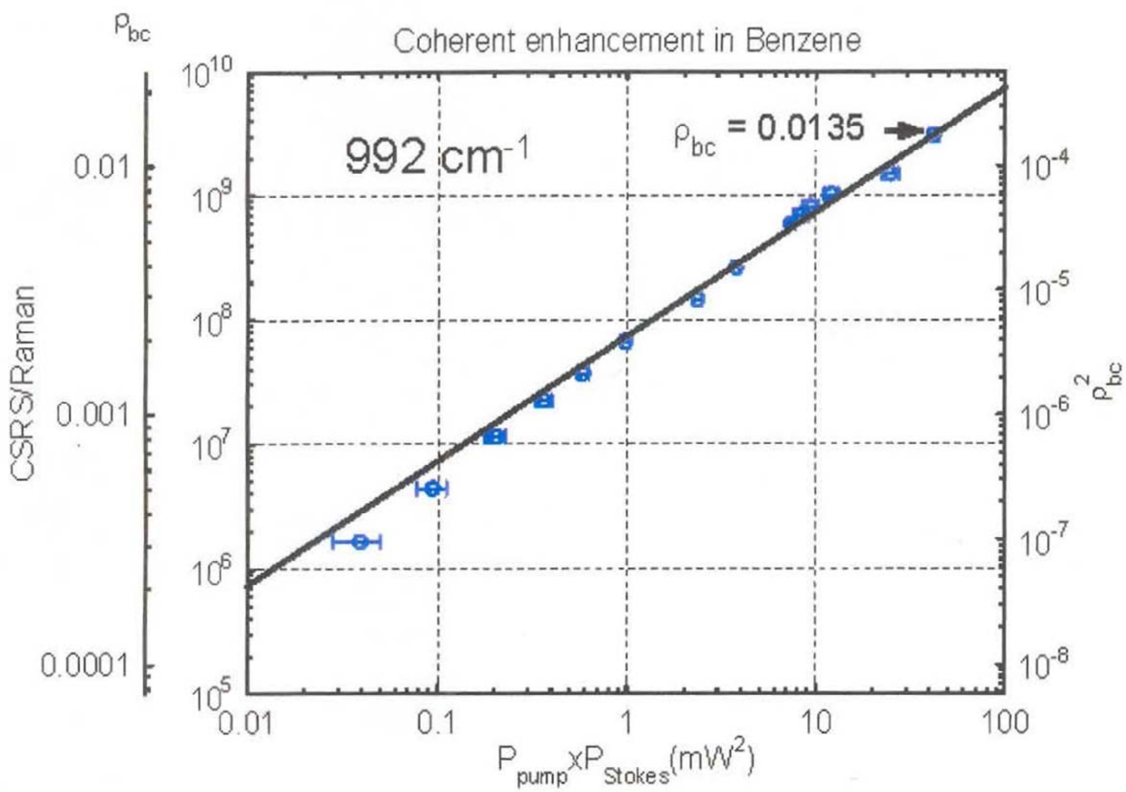
$$\langle n_s(\tau)\rangle \cong \underbrace{N\left(\frac{G\tau}{\hbar}\right)^2 \rho_{cc}}_{\text{Incoherent Raman}} + \underbrace{\frac{N(N-1)}{V}\left(\frac{G\tau}{\hbar}\right)^2 \left|\rho_{bc}\right|^2 \lambda^2 R}_{\text{Coherent Raman}}$$

Bottom Line

$$\boxed{\frac{\langle n_s\rangle_{coh}}{\langle n_s\rangle_{incoh}} \cong \frac{N}{V}\lambda^2 R \frac{\left|\rho_{bc}\right|^2}{\rho_{cc}}}$$

Cooperative Spontanous Emission

Raman "amplification" via coherence: Experimental coherence measurement



$$\frac{Coherent}{Incoherent} = \lambda^2 L \frac{N}{V} \frac{4\pi}{\Omega} \rho_{bc}^2$$

ρ_{bc} , experimental = 0.0135

ρ_{bc} , theoretical = 0.018

- 9-10 orders of magnitude enhancement over spontaneous Raman
- High directionality
- Maximum theoretical coherence: 1/2

Science
SCIENCE VOL 316 13 APRIL 2007

Optimizing the Laser-Pulse Configuration for Coherent Raman Spectroscopy

Dmitry Pestov,[1*] Robert K. Murawski,[1,2] Gombojav O. Ariunbold,[1] Xi Wang,[1] Miaochan Zhi,[1] Alexei V. Sokolov,[1] Vladimir A. Sautenkov,[1] Yuri V. Rostovtsev,[1,2] Arthur Dogariu,[2] Yu Huang,[2] Marlan O. Scully[1,2]

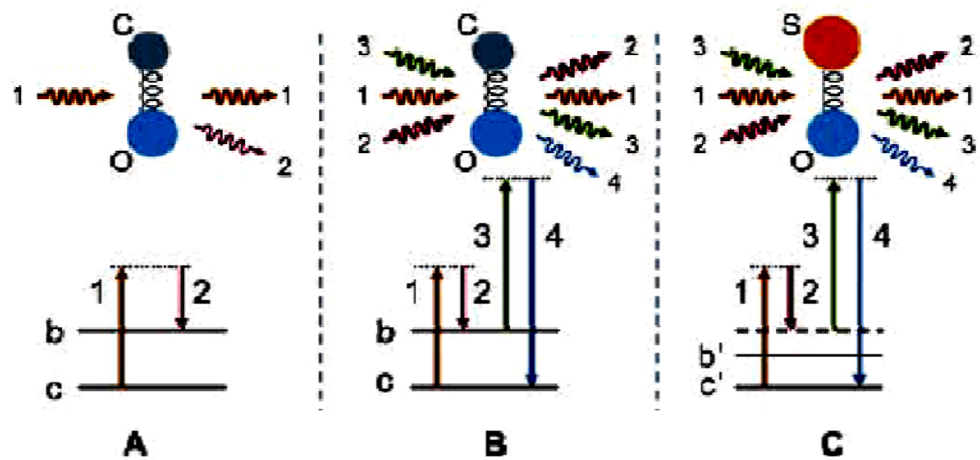
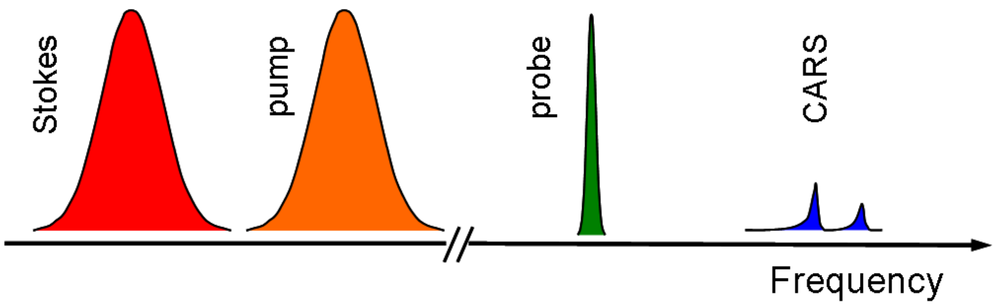
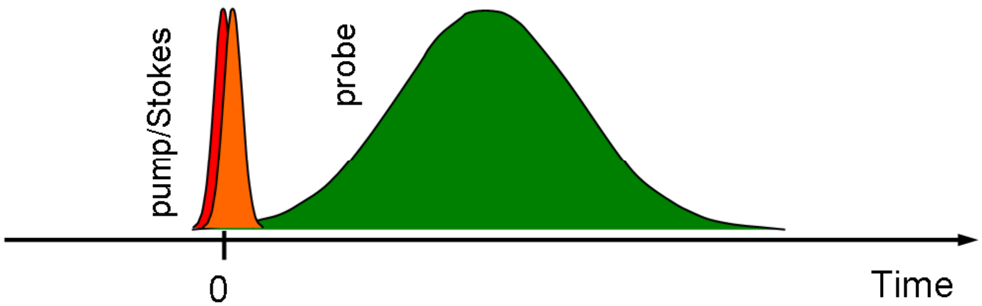


Fig. 1. Level diagram and schematic of different scattering processes on simple molecules. In this example, CO is a target molecule and SO is a background molecule. **(A)** Incoherent Raman scattering (pulse 2) was derived from laser pulse 1 scattering off of the CO molecule. **(B)** CARS signal 4 was derived from probe pulse 3 scattering off of the CO molecular vibration, coherently prepared by pulses 1 and 2. **(C)** One of the possible channels for the NR background generation in SO. c, the ground state of the CO molecule; b, the target vibrational state of the CO molecule; c′, the ground state of the background molecule; b′, an off-resonant vibrational state of the background molecule.

Optimal CARS Technique



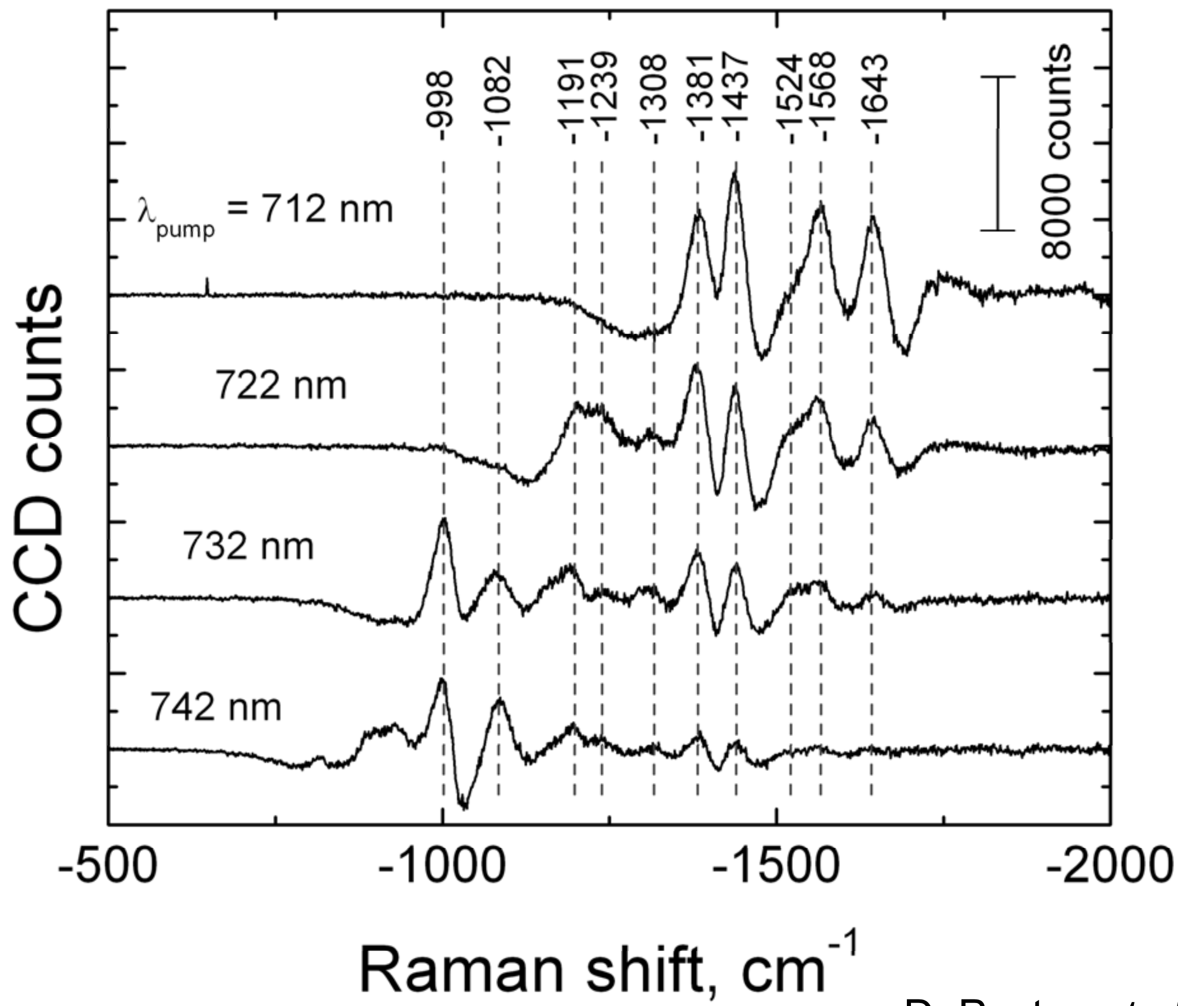
☺ Efficient broadband excitation and frequency-resolved probing, multi-channel detection => comprehensive species-specific information.



☺ The technique is relatively insensitive to signal amplitude fluctuations.

☺ Mitigation of non-resonant FWM and straightforward discrimination of the resonant response against the non-resonant one.

Backscattered CARS on spores



Parameters:
Pump
712-742 nm,
2.0 μJ/pulse

Stokes
803 nm,
3.9 μJ/pulse

Probe
577.9 nm,
FWHM~0.7 nm
~0.5 μJ/pulse

Int. time is 2 min.

D. Pestov *et al.*, *Science* **316**, 265 (2007).

Take-Home Message

FAST CARS is a promising technique that has a great potential for real-time detection and chemically-selective imaging applications.

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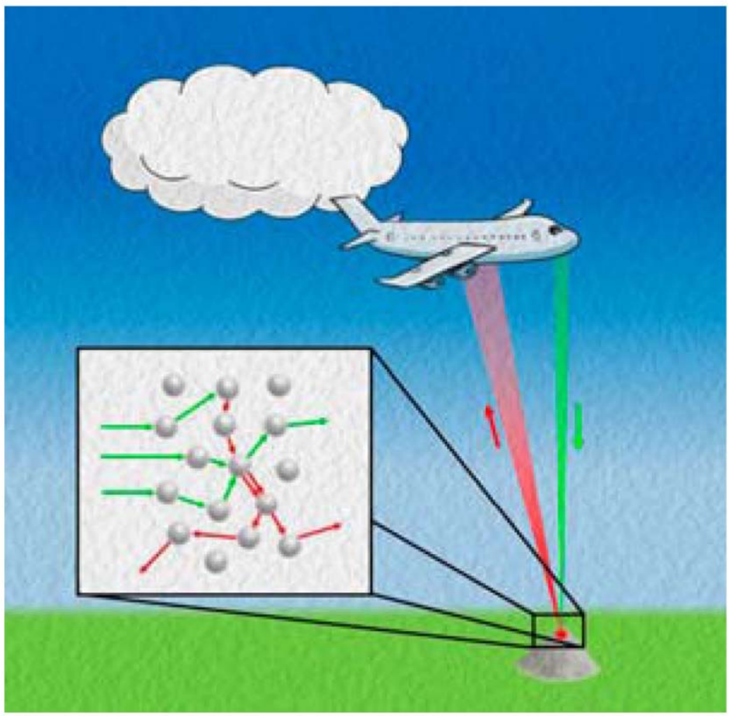
Photon pinball identifies chemicals from afar

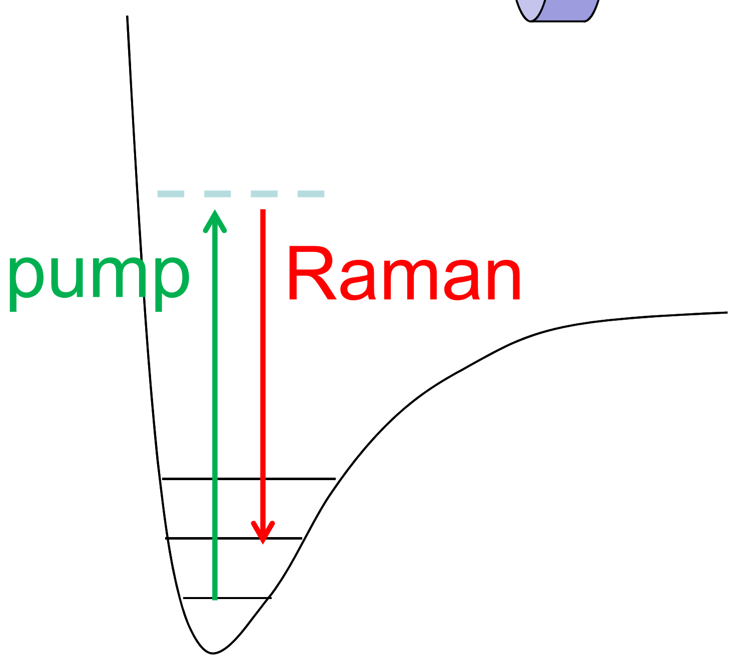
13 August 2014 Andy Extance



Photons bouncing through a powder can create a 'random laser', also amplifying Raman scattering © NAS

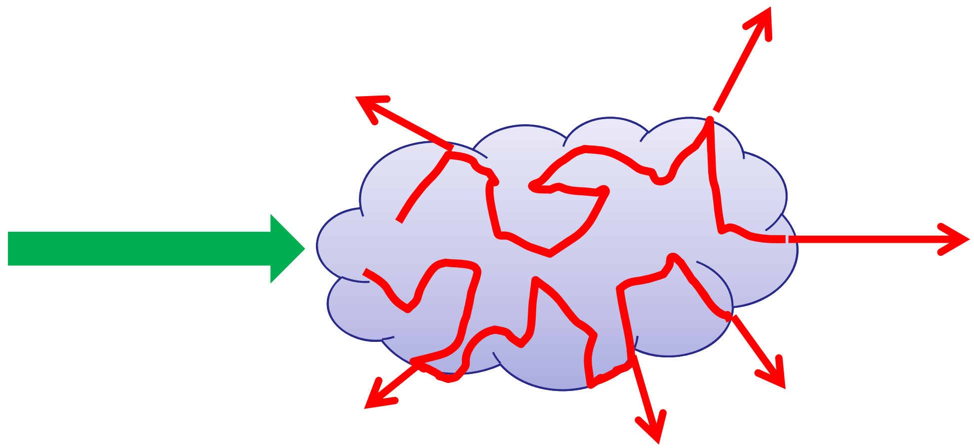
US scientists have pushed the range at which chemicals can be remotely identified beyond a kilometre by turning the samples themselves into lasers. By triggering bright random Raman laser emission, Marlan Scully and Vladislav Yakovlev from Texas A&M University and their team successfully distinguished a series of similar white powders.

What is a **Raman laser**?



Raman medium converts green pump photons to red Raman laser. Cavity contains red light.

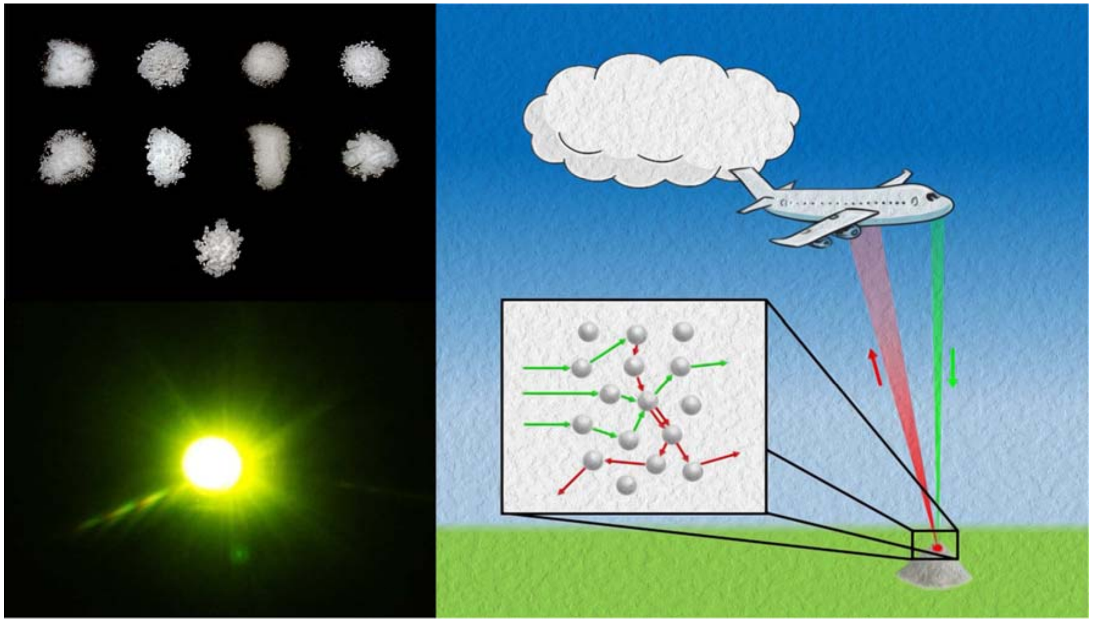
What is a **random Raman laser**?



no
cavity

Raman powder converts green pump photons to red Raman laser.
Scattering powder matrix contains red light.

Random Raman Laser



TAMU Team: ‘Single-shot stand-off chemical identification of powders using random Raman lasing’, News release.

The ability to remotely detect chemicals in real time at large distances opens the door to a variety of applications ranging from explosives monitoring and detection to monitoring nitrate levels for smart agriculture.

PNAS, 111 (34), 12320-12324 (2014)

Spatially offset Raman microspectroscopy of highly scattering tissue: theory and experiment

Z. Di, B. Hokr, *et al*., (TBP)

Pioneering research by Matousek et al (Applied Spectroscopy, 2005) has demonstrated the feasibility of obtaining the density of human bones in vivo using Raman Random spectroscopy. The present work extends the measurements and uses Monte Carlo simulations to further demonstrate the effect.

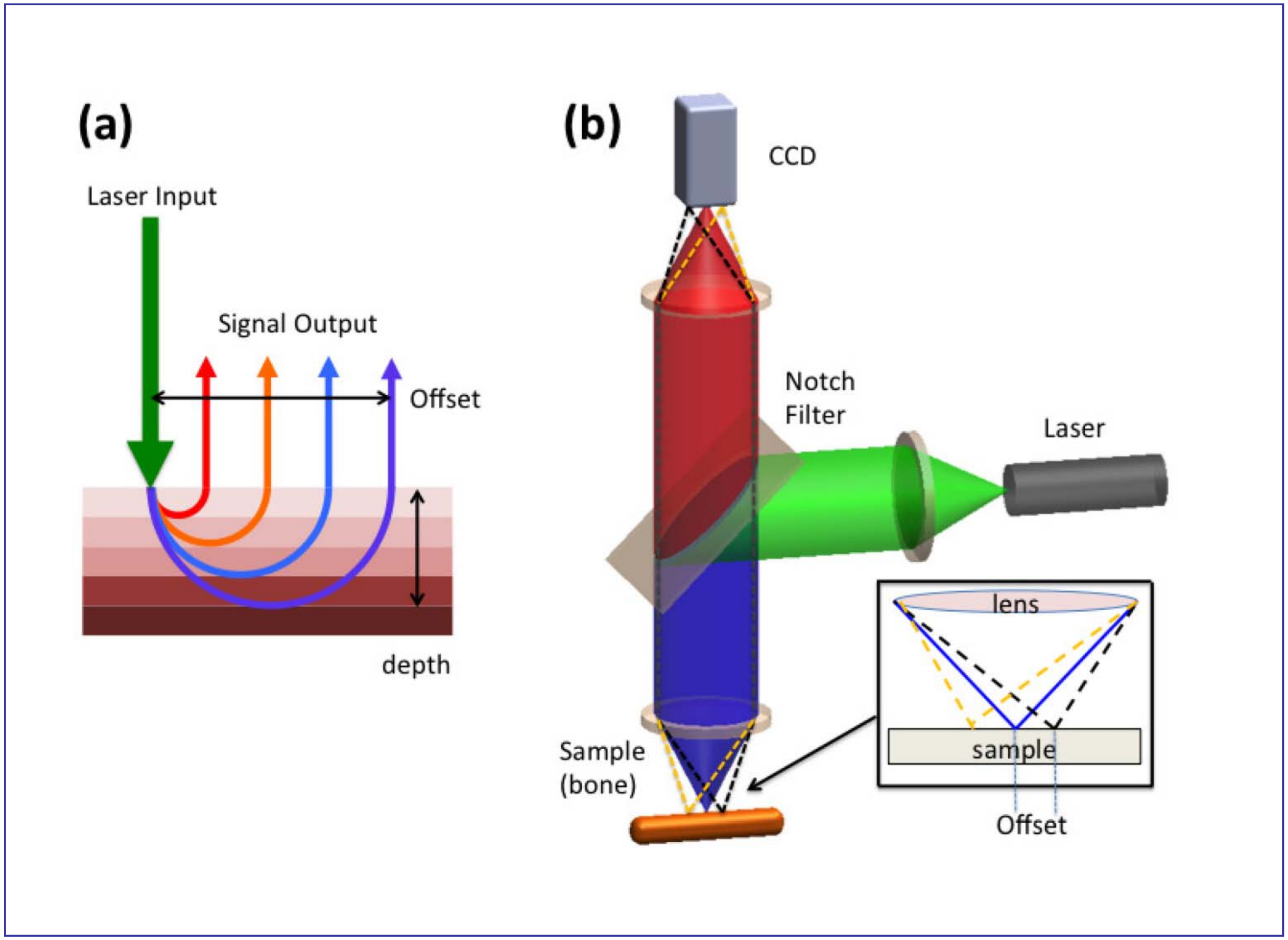


FIG. 1: Conceptual figure for (a): spatially offset Raman spectroscopy (SORS); (b): schematic diagram of the experimental microscope Raman detection setup.

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Standoff spectroscopy via remote generation of a backward-propagating laser beam

Philip R. Hemmer, et al., Texas A&M University

In an earlier publication we demonstrated that by using pairs of pulses of different colors (e.g., red and blue) it is possible to excite a dilute ensemble of molecules such that lasing and/or gain-swept superradiance is realized in a direction toward the observer.

In the present paper, we propose a related but simpler approach on the basis of the backward-directed lasing in optically excited dominant constituents of plain air, N_2 and O_2.

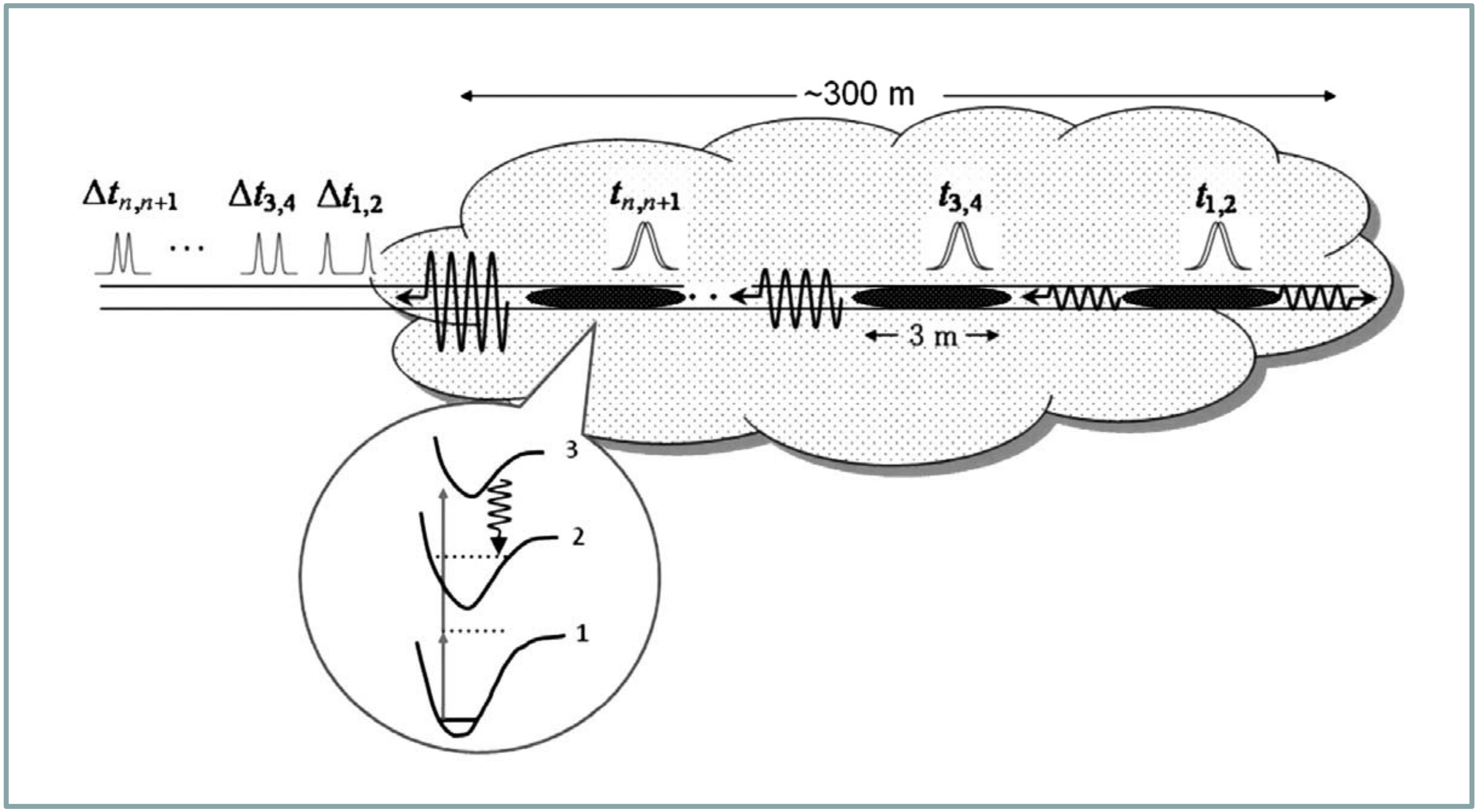

Fig. 1. SOS. Multiple pairs of pulses are generated such that the spacing between pulses in each pair is decreasing. The second pulse in each pair has a higher velocity because of atmospheric dispersion. The first pair of pulses overlaps near the back of the cloud, creating a small region of gain. Subsequent pairs overlap at closer and closer regions of the cloud, producing a swept-gain amplifier that lases back toward the observer.

Science

28 JANUARY 2011 VOL 331 SCIENCE

High-Gain Backward Lasing in Air

Arthur Dogariu, et al., Princeton University

The compelling need for standoff detection of hazardous gases and vapor indicators of explosives has motivated the development of a remotely pumped, high-gain air laser that produces lasing in the backward direction and can sample the air as the beam returns. We demonstrate that high gain can be achieved in the near-infrared region by pumping with a focused ultraviolet laser.

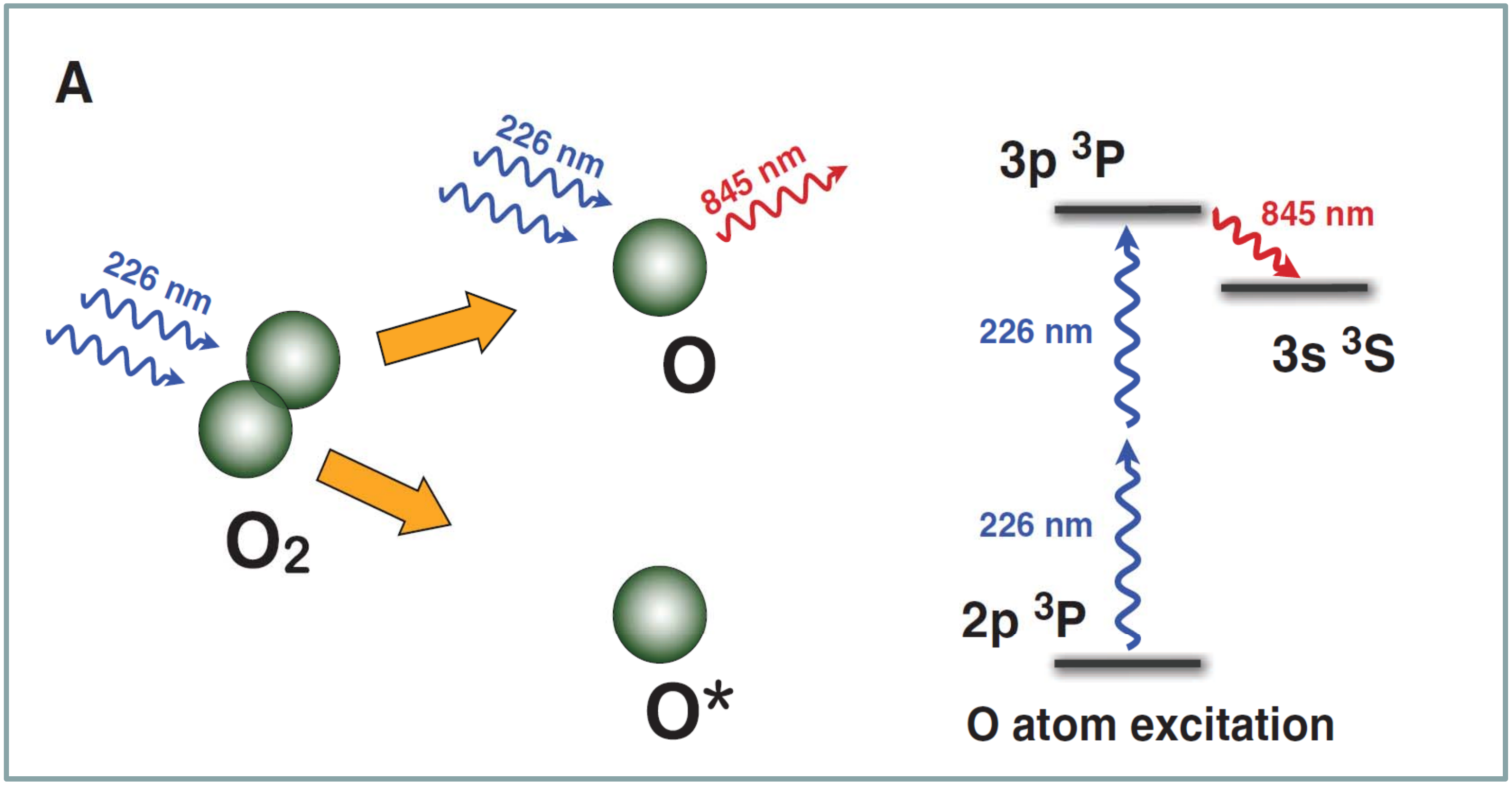

Fig. 1. (**A**) Two-photon dissociation of the oxygen molecule and subsequent two-photon resonant excitation of the ground-state oxygen atom fragment result in emission at 845 nm.

Coherence brightened laser source for atmospheric remote sensing *

Andrew Traverso, et al., Texas A&M University

"Our results suggest that the emission process exhibits atomic coherence (Dicke superradiance) in contrast with ordinary lasing where atomic coherence is negligible."

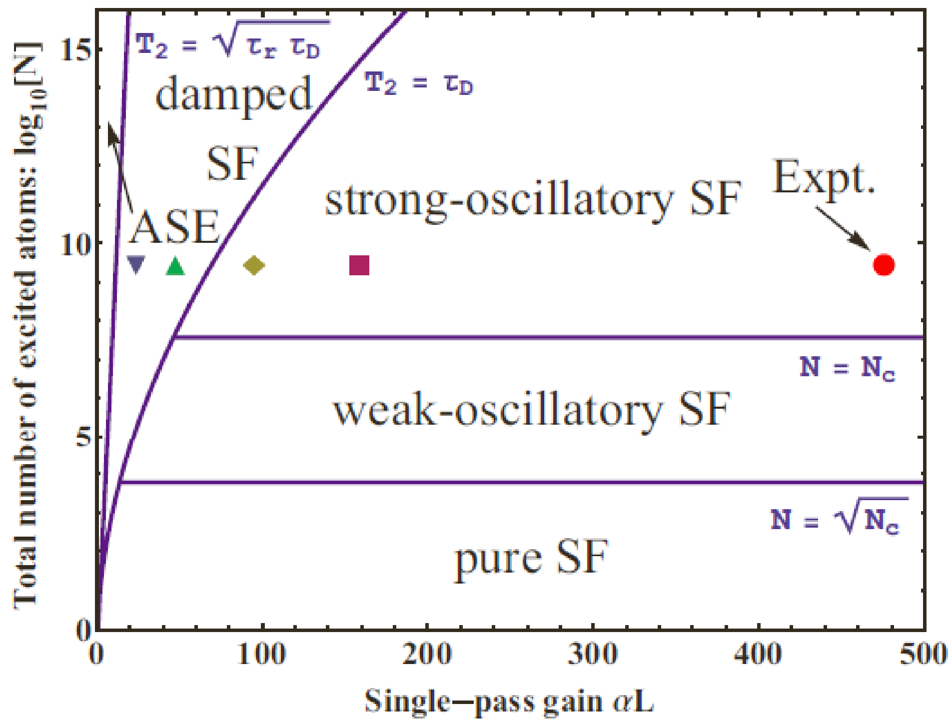

FIG. 1. (Color online) Regimes of *N*-atom cooperative spontaneous emission (adapted from Boyd et al.).

*PNAS, **109**(38),15185-15190 (2012)

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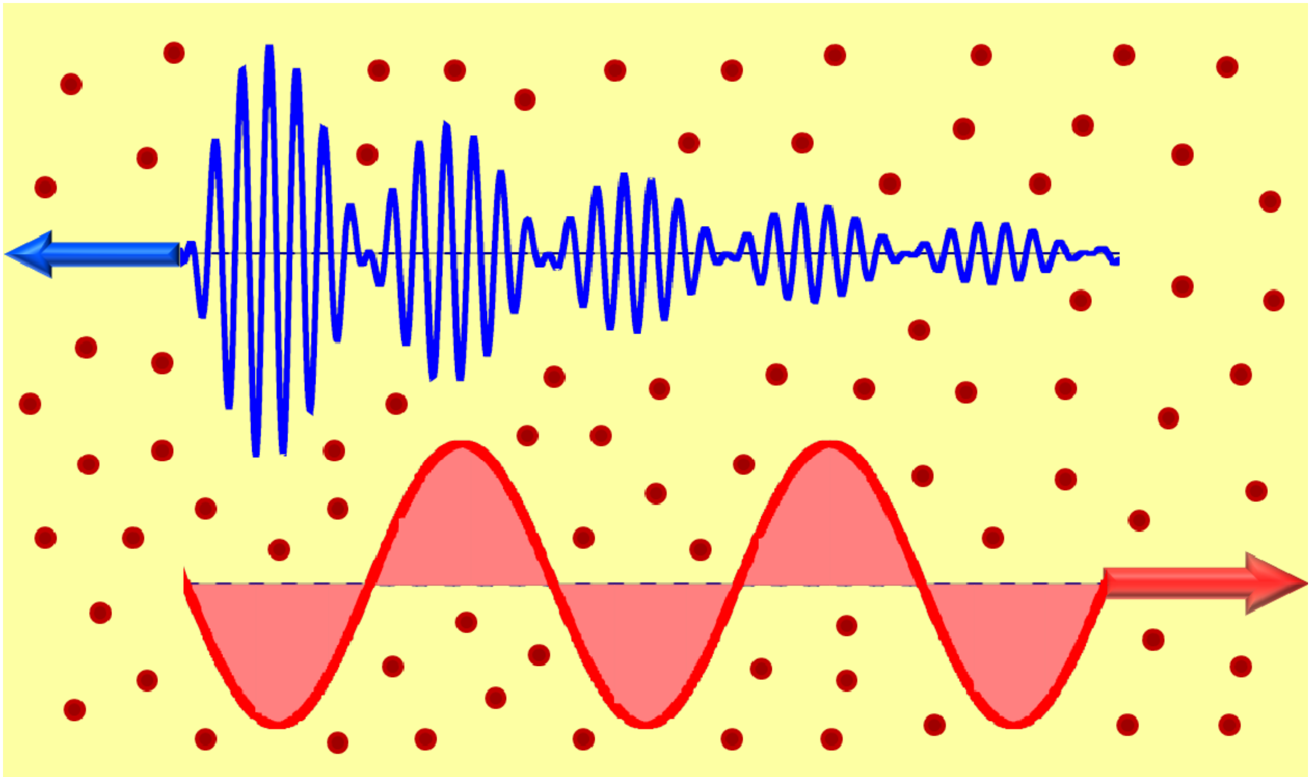
Quantum Amplification by Superradiant Emission of Radiation (QASER)



A.A. Svidzinsky, L. Yuan, M.O. Scully, **PHYSICAL REVIEW X 3, 041001, 2013**

Accepted for publication

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X

Quantum Amplification by Superradiant Emission of Radiation

Anatoly A. Svidzinsky,[1,2] Luqi Yuan,[1,2] and Marlan O. Scully[1,2,3]
[1]*Texas A&M University, College Station, Texas 77843, USA*
[2]*Princeton University, Princeton, New Jersey 08544, USA*
[3]*Baylor University, Waco, Texas 76706, USA*
(Received 18 June 2013; revised manuscript received 31 July 2013)

A laser generates light through stimulated emission of radiation and requires population inversion. Quantum interference can yield lasing without inversion. However, such phase-sensitive quantum amplification still requires some atomic population in the excited state. Here, we present a new kind of quantum amplifier based on superradiant emission of radiation (QASER) which does not need any population in the excited state!

PERSPECTIVES SCIENCE

1510 18 SEPTEMBER 2009 VOL 325

PHYSICS

The Super of Superradiance

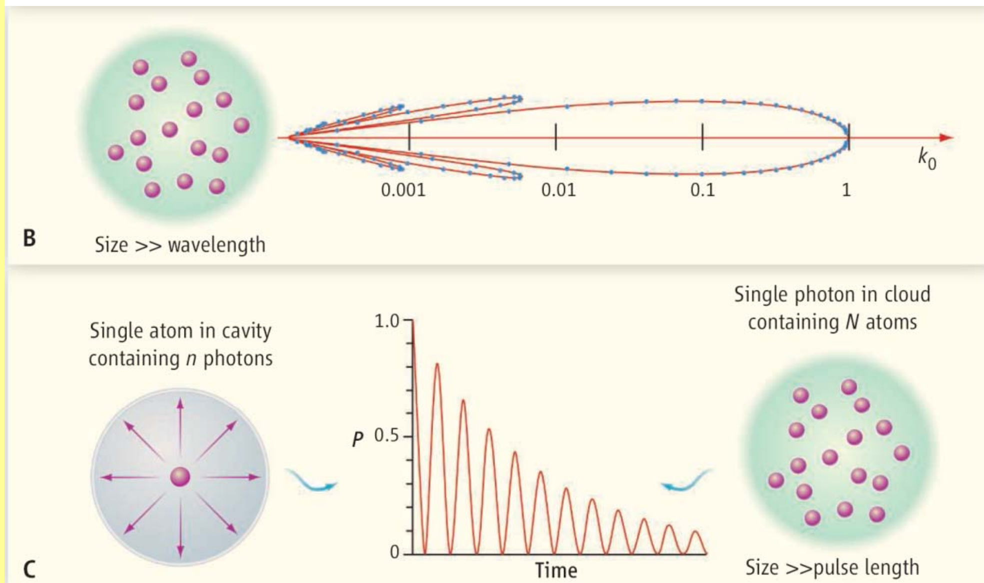
Marlan O. Scully[1,2] and Anatoly A. Svidzinsky[1]

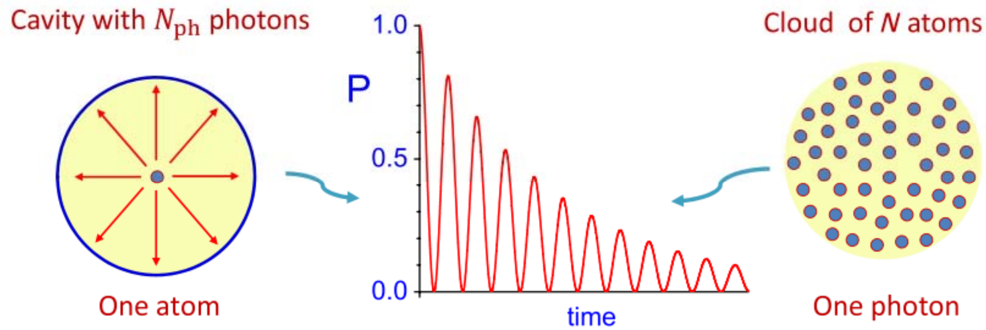
Cooperative single-photon emission from an atom ensemble will provide insights into quantum electrodynamics and applications in quantum communication.

Timed Dicke state

$$|\Psi\rangle = \frac{1}{\sqrt{N}}\left[e^{i\vec{k}_0\cdot\vec{r}_1}|\uparrow_1\downarrow_2\ldots\downarrow_N\rangle + e^{i\vec{k}_0\cdot\vec{r}_2}|\downarrow_1\uparrow_2\ldots\downarrow_N\rangle + \ldots + e^{i\vec{k}_0\cdot\vec{r}_N}|\downarrow_1\downarrow_2\ldots\uparrow_N\rangle\right]$$

A





	Probability of atomic excitation	Key variables V and N
Single atom maser	$sin^2 \frac{\wp}{\hbar} \sqrt{\frac{\hbar\nu}{\varepsilon_0 V}} \sqrt{N_{ph}}\, t$	V = photonic cavity volume N_{ph} = number of photons in cavity
Single photon superradiance	$sin^2 \frac{\wp}{\hbar} \sqrt{\frac{\hbar\nu}{\varepsilon_0 V}} \sqrt{N_{at}}\, t$	V= atomic cloud volume N_{at} = number of atoms in cloud

Can rewrite the collective superradiant oscillation frequency as:

$$\frac{\wp}{\sqrt{\hbar\varepsilon_0}} \sqrt{\frac{\hbar\nu}{\varepsilon_0 V}} \sqrt{N_{at}} = \sqrt{\frac{3}{4\pi}\lambda^2 \gamma c \frac{N_{at}}{V}} \equiv \Omega_a$$

Defining the Rabi frequency $\Omega_s(z,t) = \wp\varepsilon_s(z,t)/\hbar$ we have our working equation:

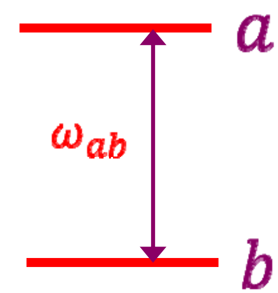
$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\Omega_s = i{\Omega_a}^2 \rho_{ab}$$

Physics of QASER

Electromagnetic field and atoms are two coupled oscillators

Maxwell's equation for electric field

$$\left(\frac{\partial^2}{\partial t^2} - c^2\Delta\right)\Omega = -2\frac{\Omega_a^2}{\omega_{ab}}\frac{\partial^2 \rho_{ab}}{\partial t^2}$$



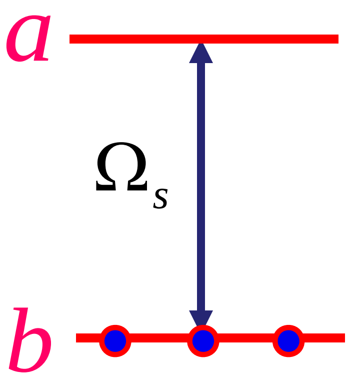
Schrodinger equation for atoms

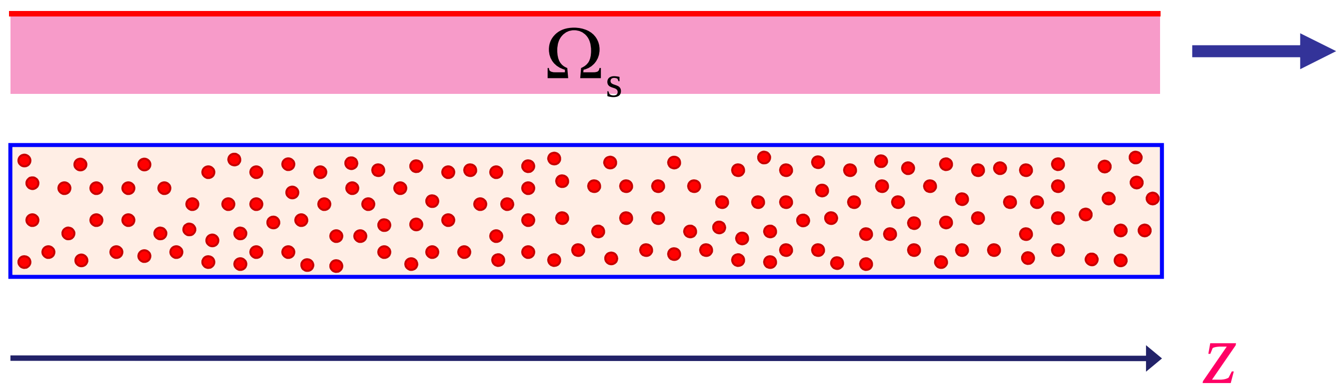
$$\frac{\partial \rho_{ab}}{\partial t} + i\omega_{ab}\rho_{ab} = i\Omega\left(\rho_{bb} - \rho_{aa}\right)$$

Coupling between oscillators is nonlinear

Driving of atoms modulates coupling strength

Light amplification with no population in excited state





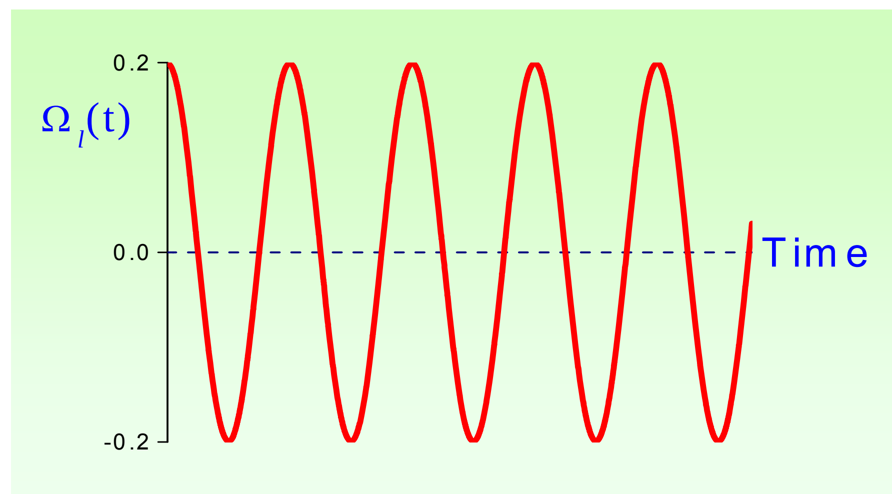
For uniform lasing field we obtain:

$$\frac{\partial^2 \Omega_s}{\partial t^2} + \Omega_a^2 (\rho_{bb} - \rho_{aa}) \Omega_s = 0$$

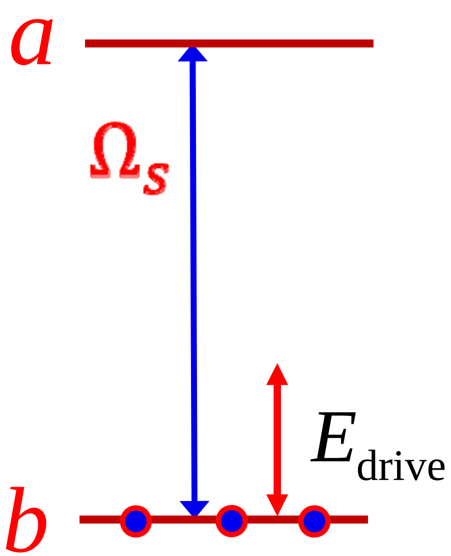
$$\Omega_a = \sqrt{\frac{3cN\lambda^2\gamma}{8\pi}}$$

Laser field oscillates with collective frequency:

$$\Omega_s(t) = \Omega_s(0)\cos\left(\Omega_a\sqrt{\rho_{bb} - \rho_{aa}}\, t\right)$$



What if we periodically change population of level *a* or *b* ?

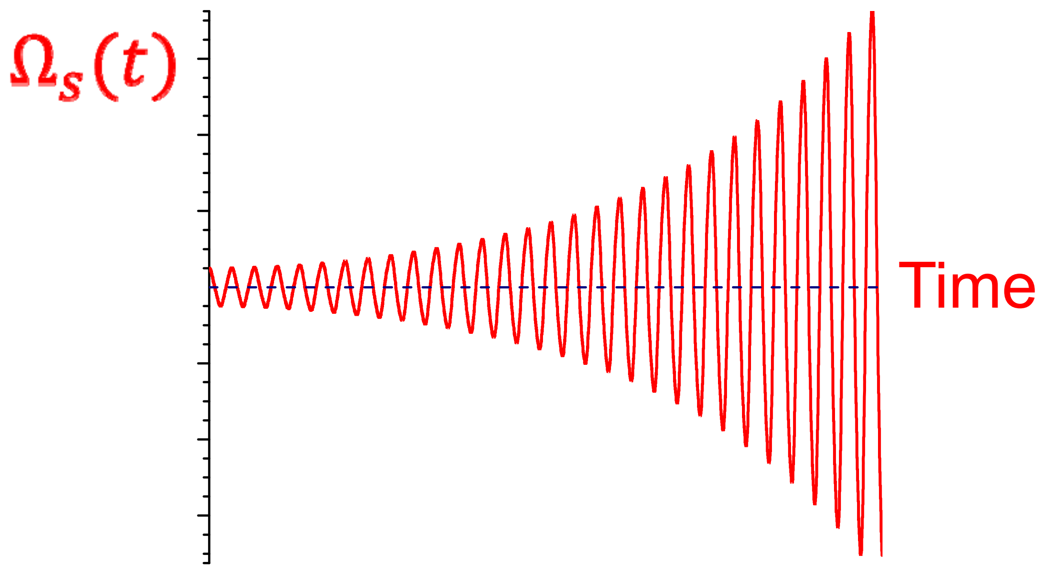


$$\frac{\partial^2 \Omega_s}{\partial t^2} + \Omega_a^2 \left(\rho_{bb} - \rho_{aa}\right) \Omega_s = 0$$

↓ **Mathieu equation**

$$\frac{\partial^2 \Omega_s}{\partial t^2} + \Omega_a^2 \left[1 - \delta \cos(2\nu_d t)\right] \Omega_s = 0 \qquad \delta << 1$$

If $\nu_d = \frac{\Omega_a}{m}$, *m*=1,2,3,… then field would exponentially grow with time:

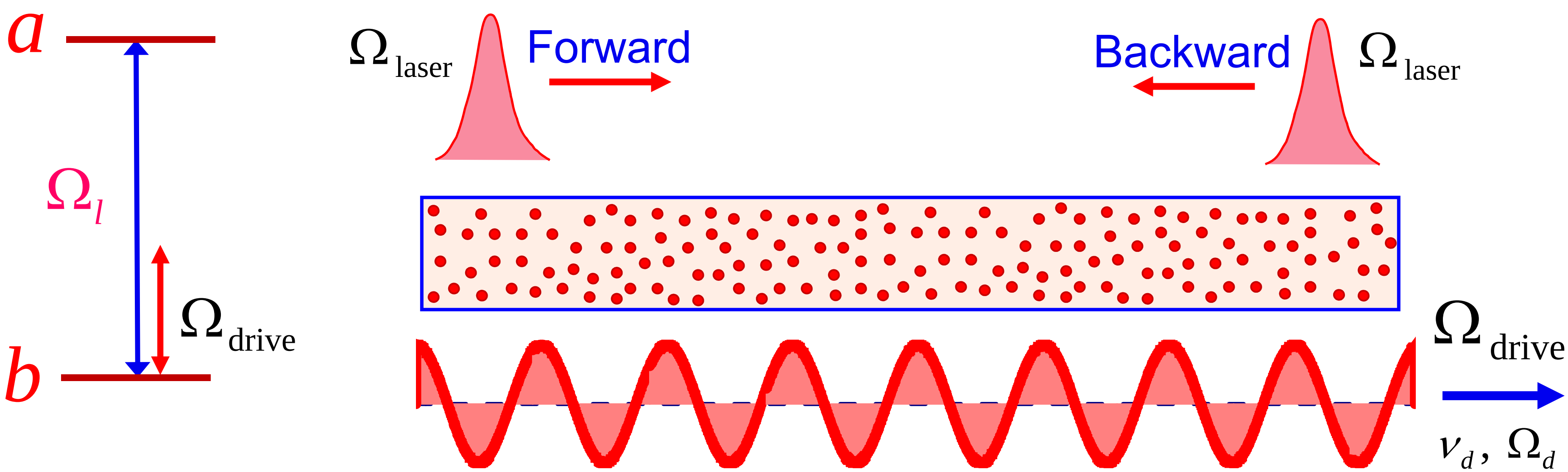


For 1st order resonance:

$$\Omega_s(t) \propto e^{Gt}, \quad G = \frac{\delta \cdot \Omega_a}{8}$$

G – gain per unit time

Propagating driving field: general analysis



Forward direction:

$G=0$

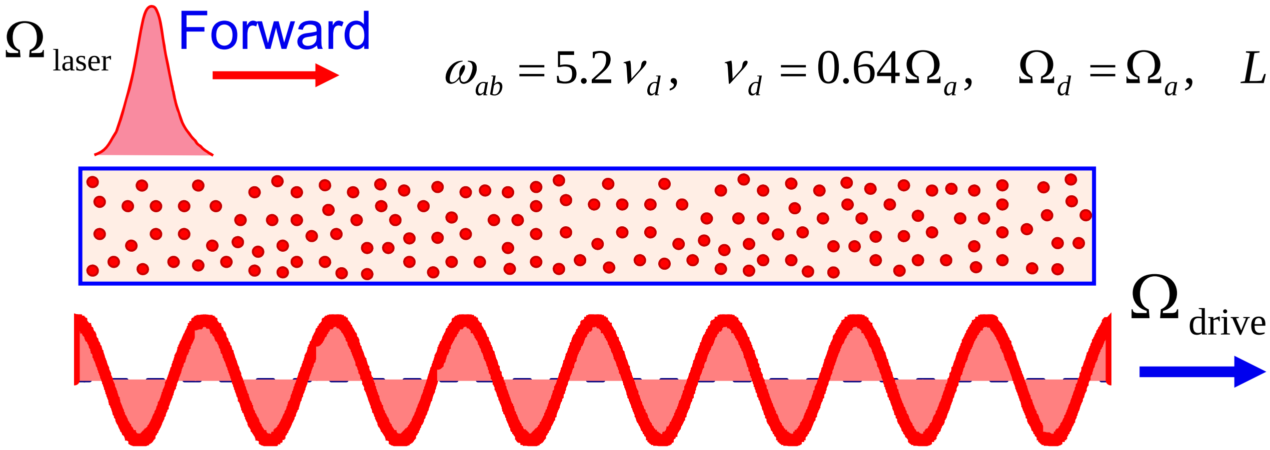
Backward direction:

If Stark shift is suppressed then

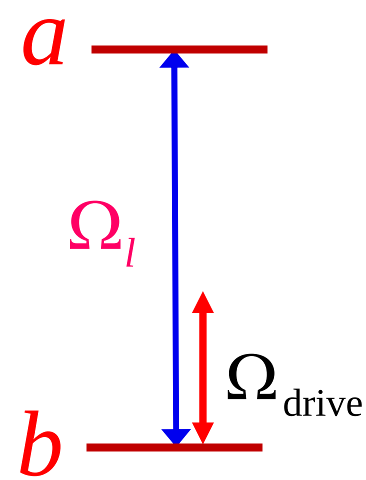
there is gain for $\nu_d > \frac{\Omega_a}{\sqrt{2}}$

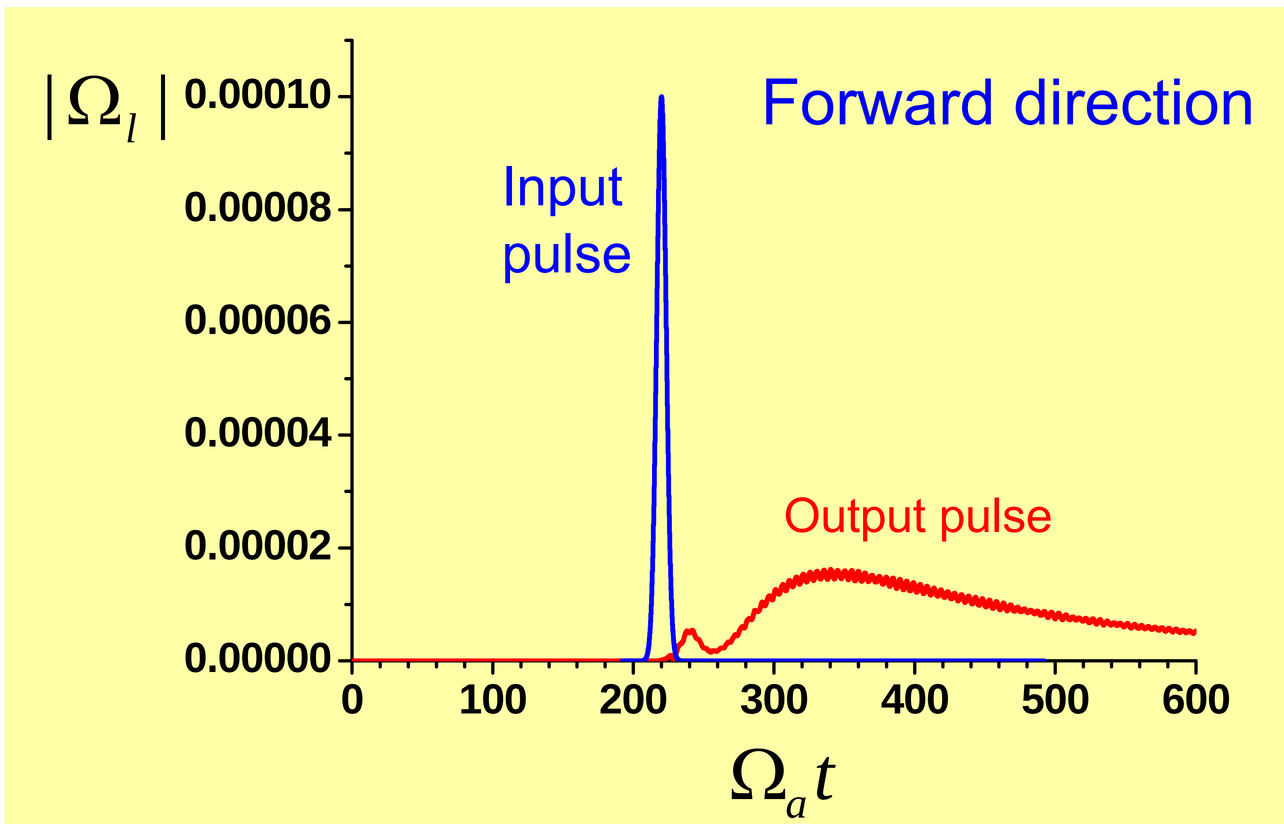
Gain per unit time: $G_t = \frac{\Omega_a}{3\sqrt{2}}\left(\frac{\Omega_d}{\omega_{ab}}\right)^2$

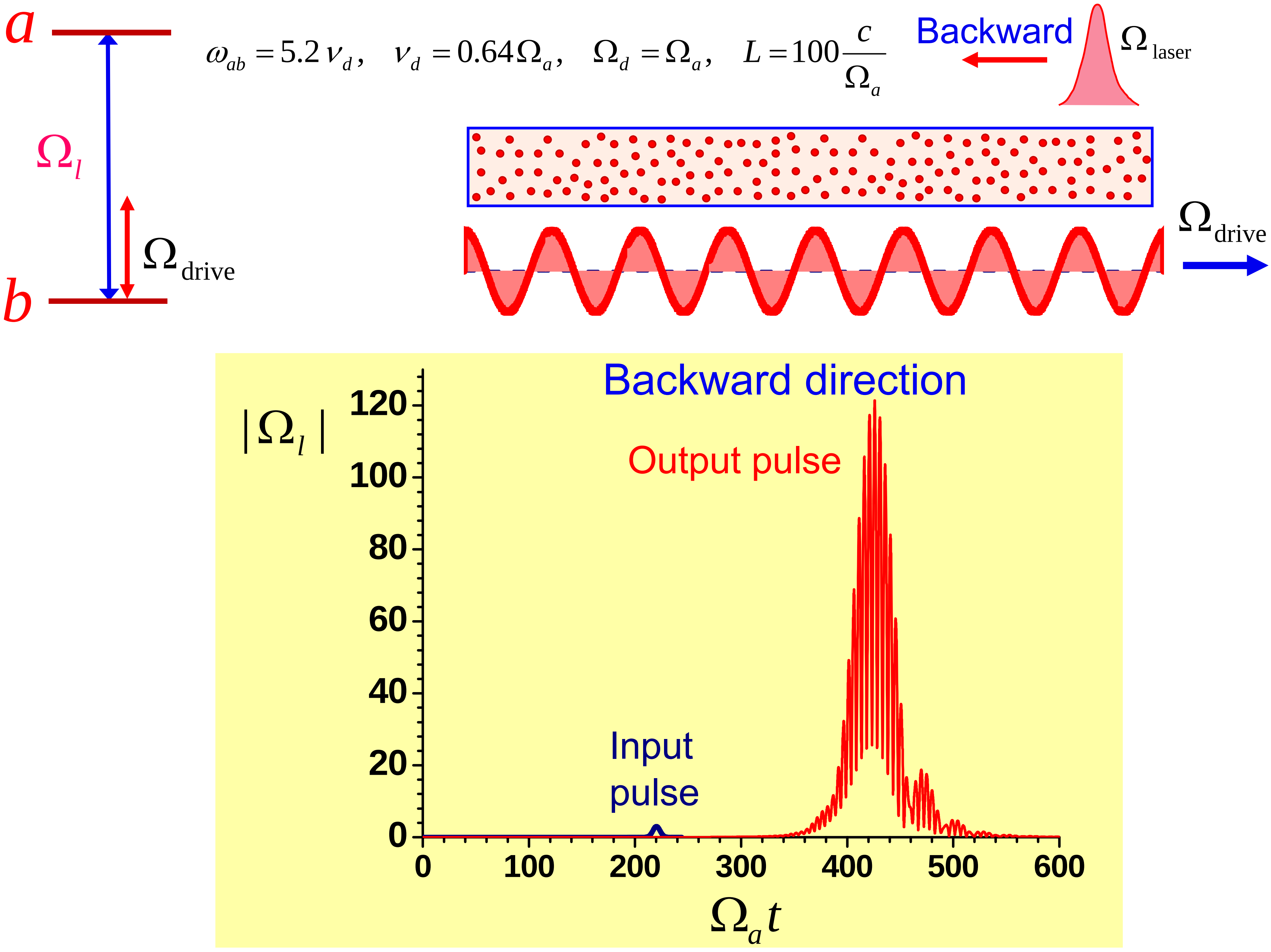
Numerical solution of Maxwell-Schrödinger equations



$$\omega_{ab} = 5.2\,\nu_d, \quad \nu_d = 0.64\,\Omega_a, \quad \Omega_d = \Omega_a, \quad L = 100\frac{c}{\Omega_a}$$

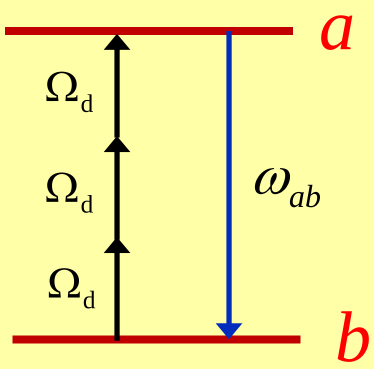





a
b
Ω_l
Ω_{drive}
$\omega_{ab} = 5.2\,\nu_d, \quad \nu_d = 0.64\Omega_a, \quad \Omega_d = \Omega_a, \quad L = 100\frac{c}{\Omega_a}$
Backward
Ω_{laser}
Ω_{drive}
Backward direction
$|\Omega_l|$
Output pulse
Input
pulse
0
20
40
60
80
100
120
0
100
200
300
400
500
600
$\Omega_a t$

Comparison of atomic excitation mechanisms

Multiphoton resonant excitation



Single atom phenomenon

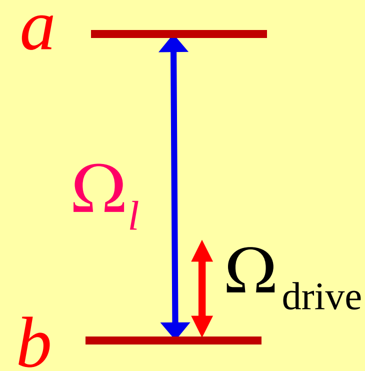
Multiphoton resonance with atomic transition frequency

$$\nu_d = \frac{\omega_{ab}}{m}$$

$$\rho_{aa} \approx \left(\frac{\Omega_d}{\omega_{ab}}\right)^{2m} (\omega_{ab} t)^2$$

High frequency light is emitted in the direction of driving field

Collective parametric resonance



Collective effect

Resonance with collective frequency

$$\nu_d > \frac{\Omega_a}{\sqrt{2}}$$

$$\rho_{aa} \approx |\rho_{ab}(0)|^2 \exp\left(\frac{\sqrt{2}\Omega_d^2}{3\omega_{ab}^2}\Omega_a t\right)$$

Lasing occurs in the backward direction

THE QASER REVISITED:

Insights gleaned from analytical solutions to simple models*

Marlan O. Scully

Princeton University, Princeton, NJ 08544,

Texas A&M University,

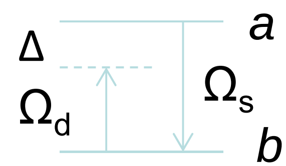
College Station,TX 77843,

Baylor University, Waco, TX 76798

Lasers and masers typically require population inversion. But with phase coherent atoms(phasers), we get lasing without inversion (e.g., 10% of the atoms excited). However in recent work we found that it is possible to get coherent light emitted with no atoms excited, via Quantum Amplification of Superradiant Emission of Radiation(QASER). In particular, we found that utilizing collective superradiant emission, we can generate coherent light at high frequency in the UV or x-ray bands by driving the atomic system with a low-frequency (e.g., infrared) source. We here present a simple analysis based on near resonant QASER operation and on a multi-photon Hamiltonian obtained by, e.g., a canonical transformation.

June 20, 2014

SIMPLE GAIN EQUATION



$$\dot{\Omega}_s = -i\Omega_a^2 \rho_{ab}$$

$$\dot{\rho}_{ab} = -i\Omega_s (1 - 2\rho_{aa})$$

Photon Osc. →

$$\ddot{\Omega}_s = (-i)^2 \Omega_a^2 (1 - 2\rho_{aa}) \Omega_s$$

Atomic Osc. →

$$\ddot{\rho}_{ab} = (-i)^2 \Omega_a^2 (1 - 2\rho_{aa}) \rho_{ab}$$

$$\ddot{X} = -\Omega_a^2 \left(1 - \left(\frac{\Omega_d}{\Delta} \right)^2 \cos 2\Delta t \right) X$$

$$X = \Omega_s \text{ or } \rho_{ab}$$

SAME EQUATION SAME GAIN

$$\text{Gain} = \frac{\Omega_a}{4} \left(\frac{\Omega_d}{\Delta} \right)^2$$

Parametric harmonic oscillator (Mathieu equation)

$$\frac{\partial^2 x}{\partial t^2} + \Omega_a^2 \left[1 - \delta \cos(\nu_d t)\right] x = 0 \qquad \delta << 1$$

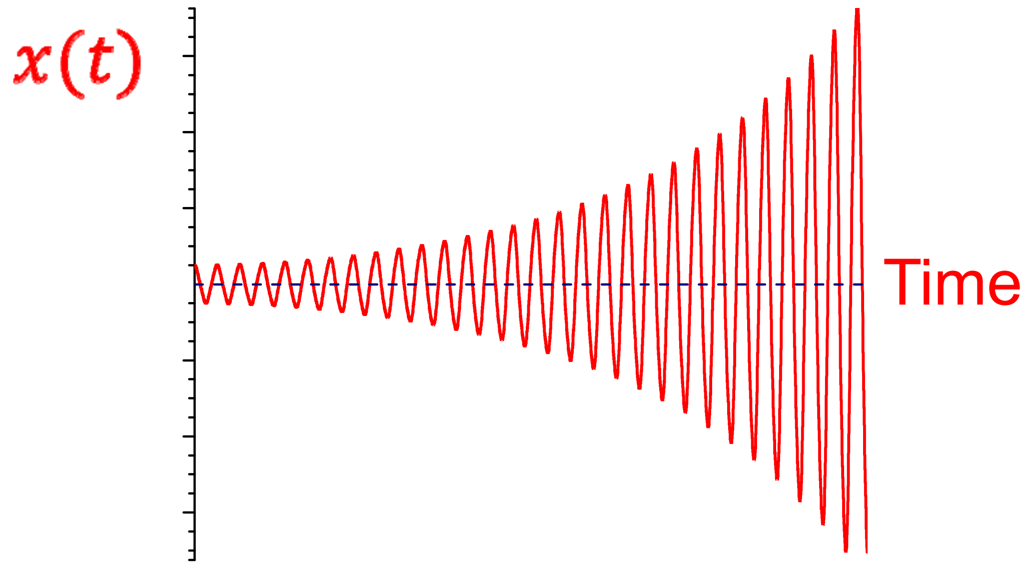
Parametric resonance

If $\nu_d = \dfrac{2\Omega_a}{m}$, m=1,2,3,… then oscillations exponentially grow with time:

For 1st order resonance:

$$x(t) \propto e^{Gt}, \quad G = \frac{\delta \cdot \nu_d}{8}$$

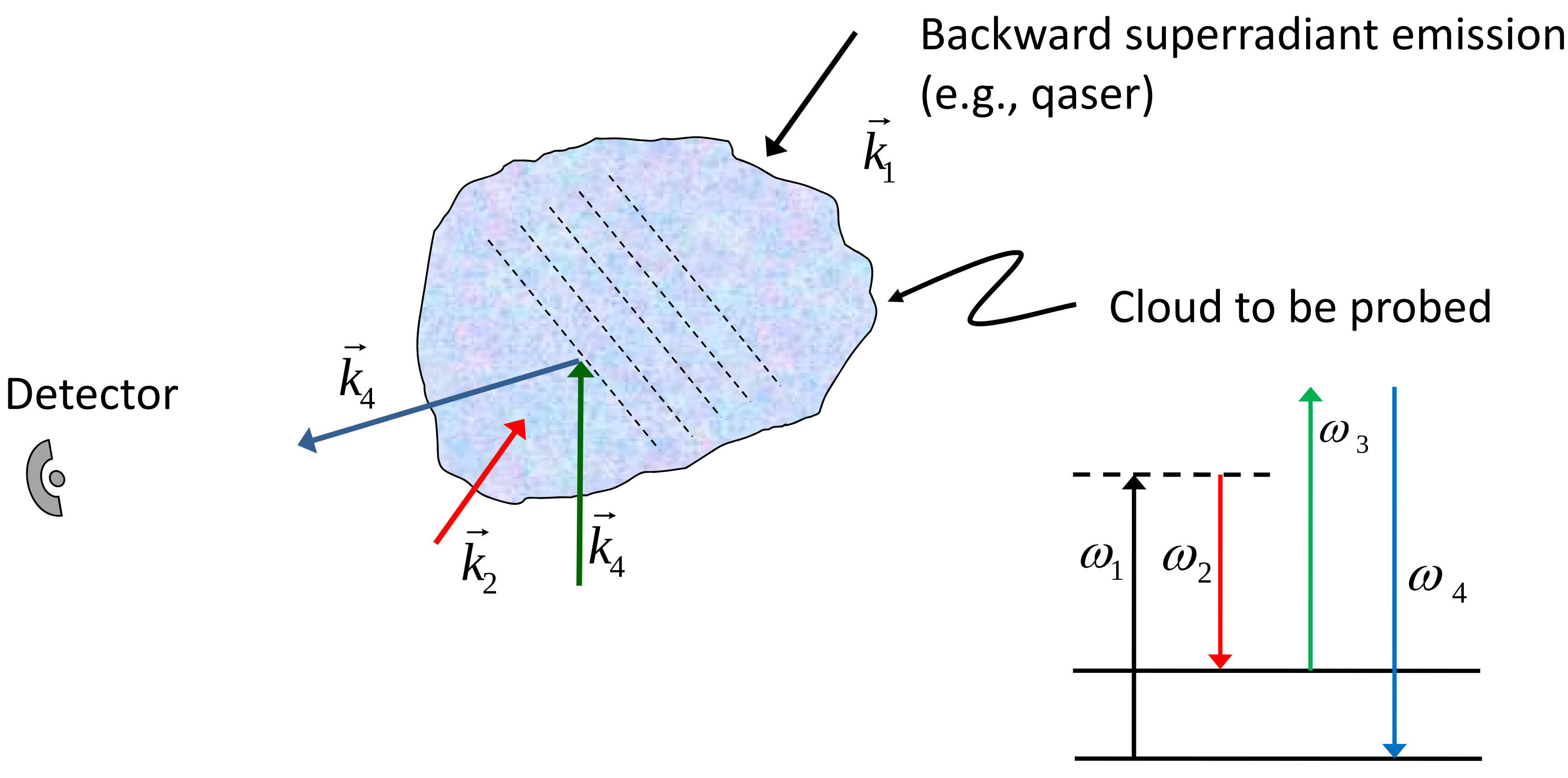
G – gain per unit time



QASER Summary

We found a new way to achieve light amplification in the backward direction with no population in excited state by means of collective parametric resonance.

CARS In The Sky*



*Yuan, et al., Laser Phys. Lett. (2011)

“SUMMARY”

$$\frac{\langle n_s \rangle_{coh}}{\langle n_s \rangle_{incoh}} \cong \frac{N}{V} \lambda^2 R \frac{|\rho_{bc}|^2}{\rho_{cc}}$$

RAMAN AT RANGE

1) Anthrax Detection via FAST CARS
   a) Motivation
   b) Coherent vs. Spontaneous Raman (Boyd, Shen, Welch)
2) Random Raman
   a) Remote Chemical Detection
   b) Bone Density Measurements

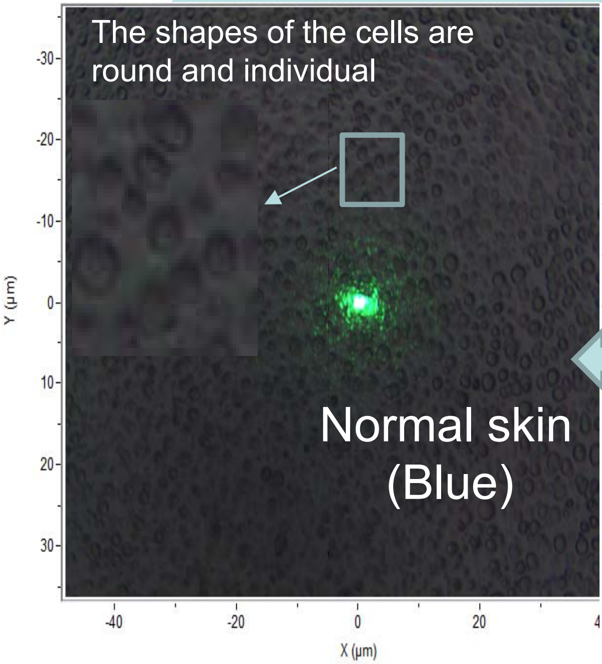
$$E_{\text{out}} \approx E_{\text{in}} \exp\left(\frac{\Omega_d^2}{\Delta^2} \Omega_a t \right)$$

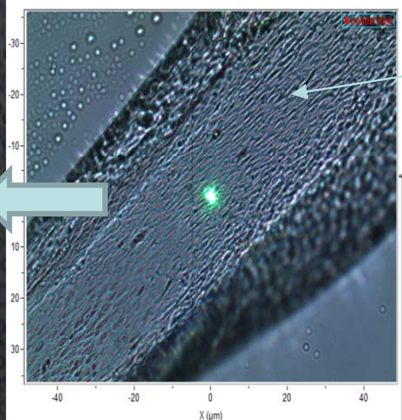
DICKE AT A DISTANCE

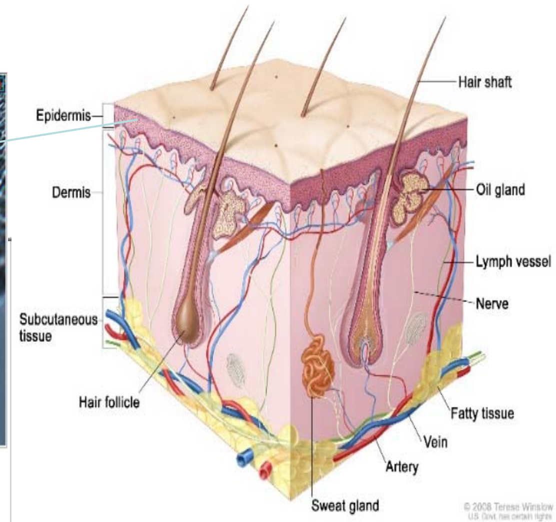
3) Superradiant Swept Gain
   a) High Gain Backward Lasing
   b) Coherence Brightened Air Laser
4) Backward Quantum Amplification by Superradiant Emission of Radiation
   a) Concept and Numerical Simulation
   b) Simple Gain Calculation
5) CARS in the Sky (via S. L. Chin on the ground)
6) Summary

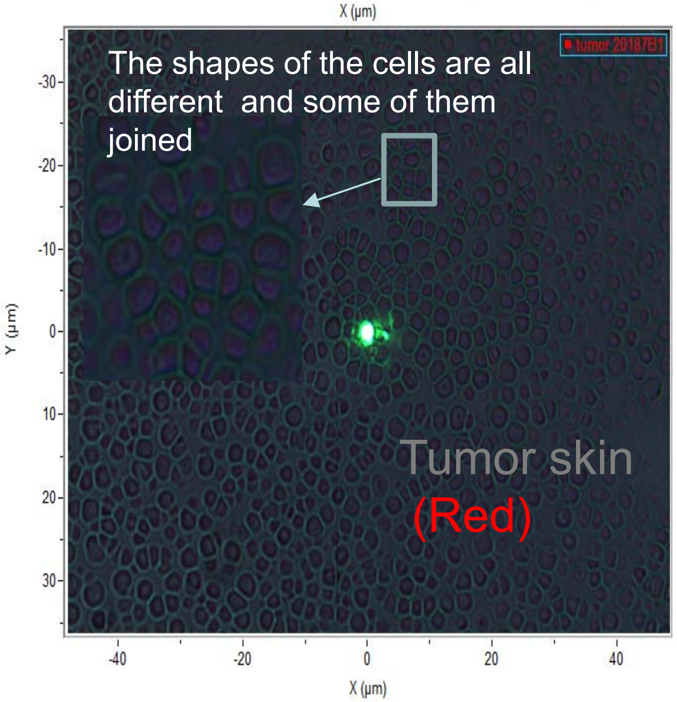
A Raman Spectroscopic Study of skin tumors of Sinclair Piglets

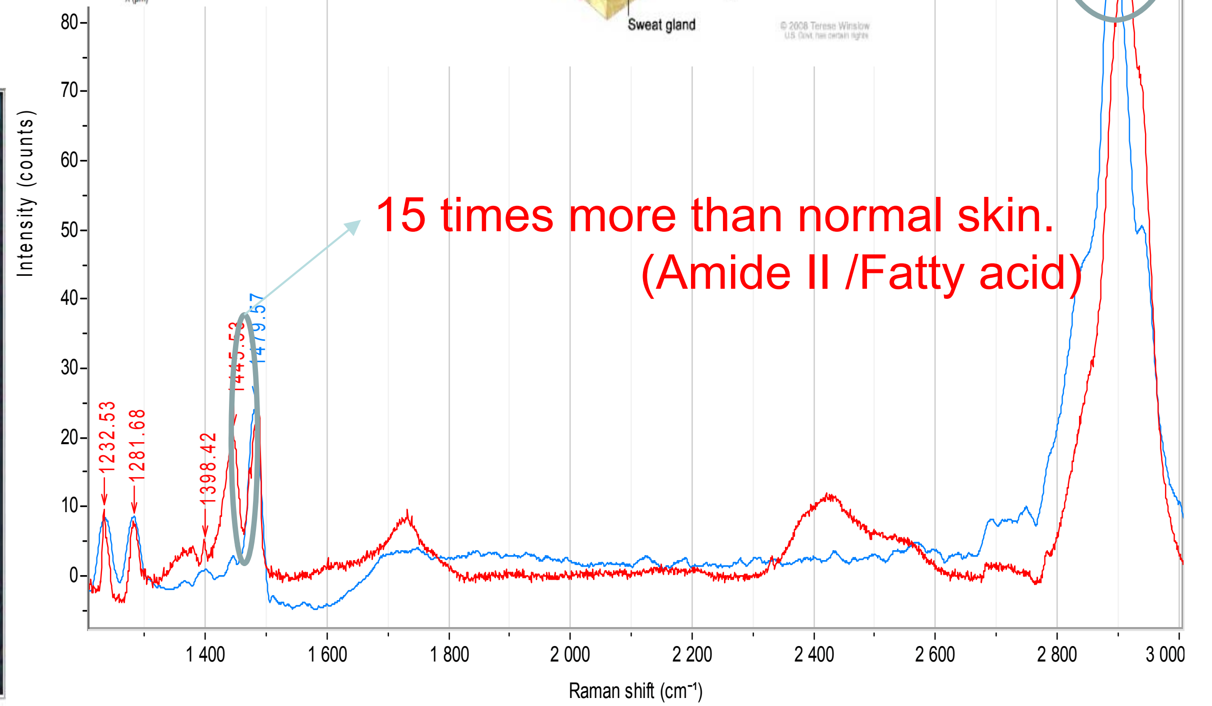
Main results on normal skin vs. tumor skin slides









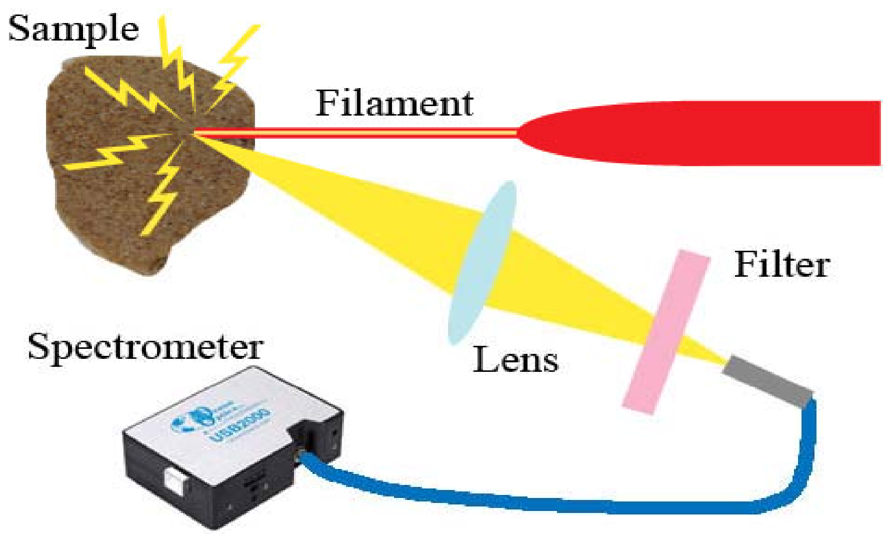


10/22/14, Nara

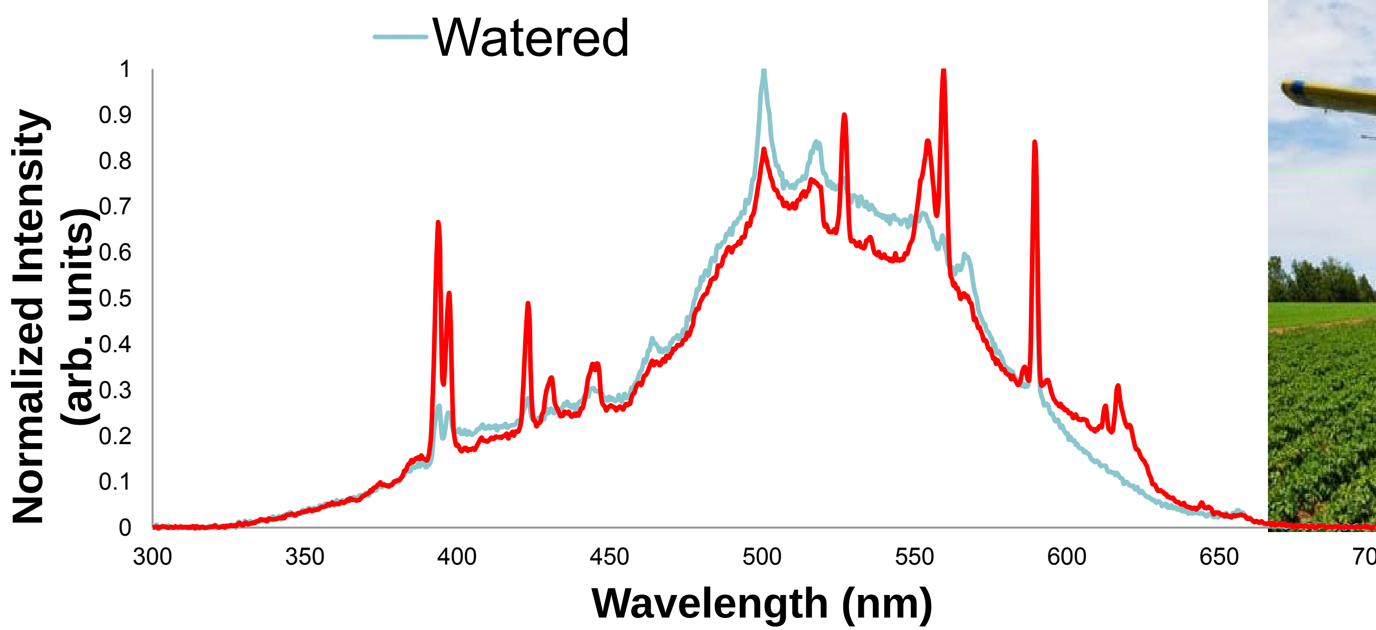
Filament-induced breakdown spectroscopy (FIBS)

Aleksei Zheltkiov, Texas A&M, Moscow State

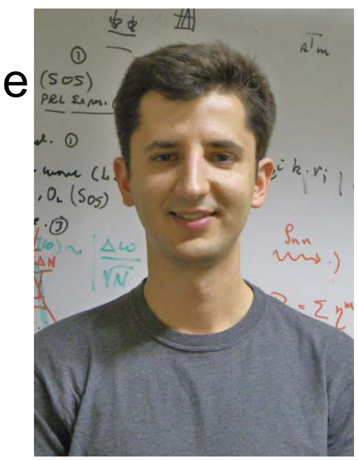
- Applications in precision agriculture, military, all areas of remote sensing
- Remotely detect and identify biological and mineralogical samples



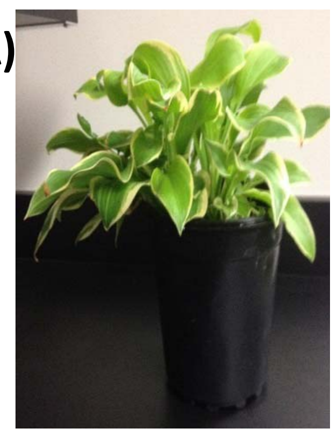
Laser-Induced Breakdown Spectroscopy (LIBS) of Hosta Leaf



Dmitri Voronine
Texas A&M



A)

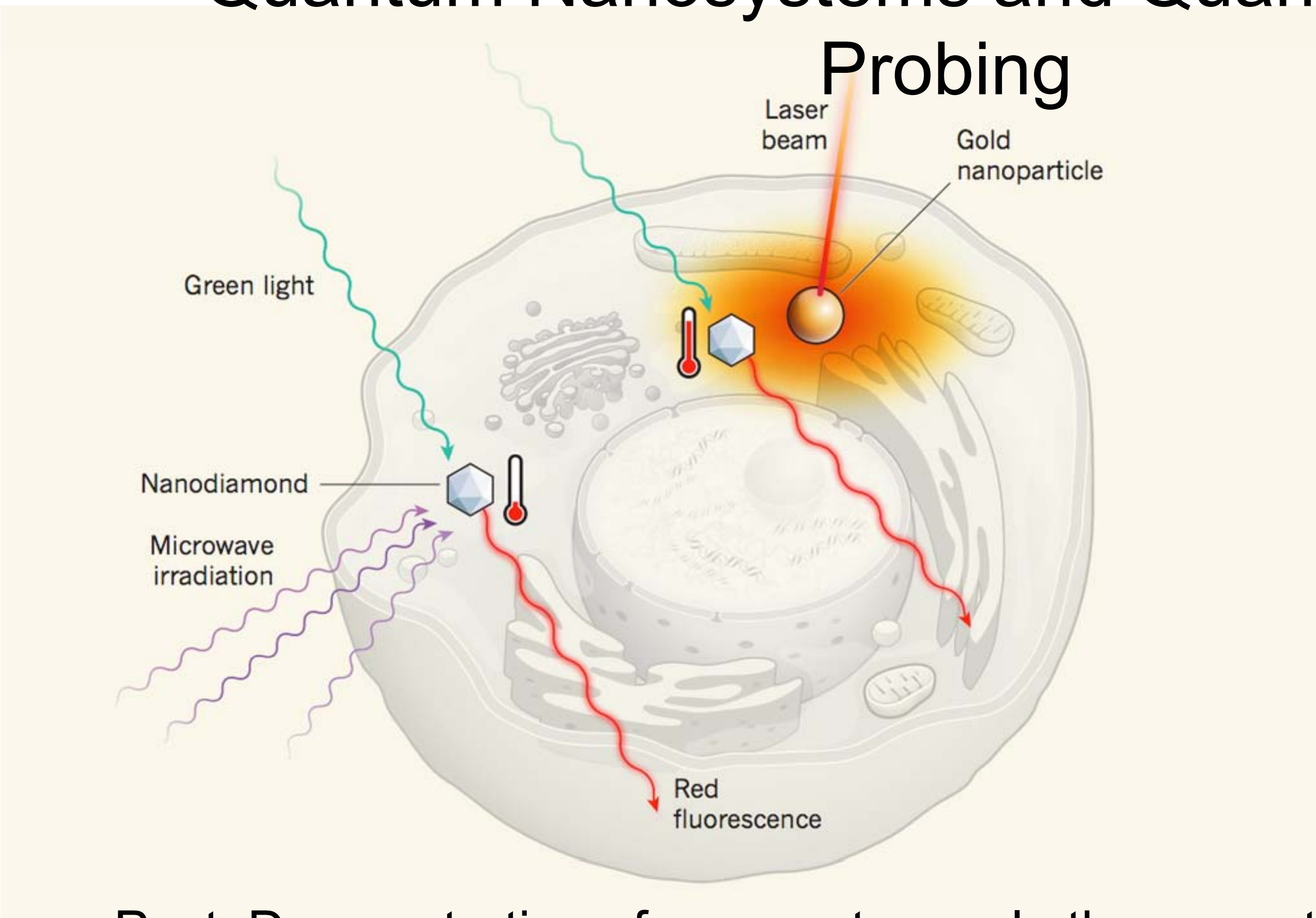


A) Hosta plant that is watered regularly.

B)

B) Hosta plant that has not been watered for 1 week.

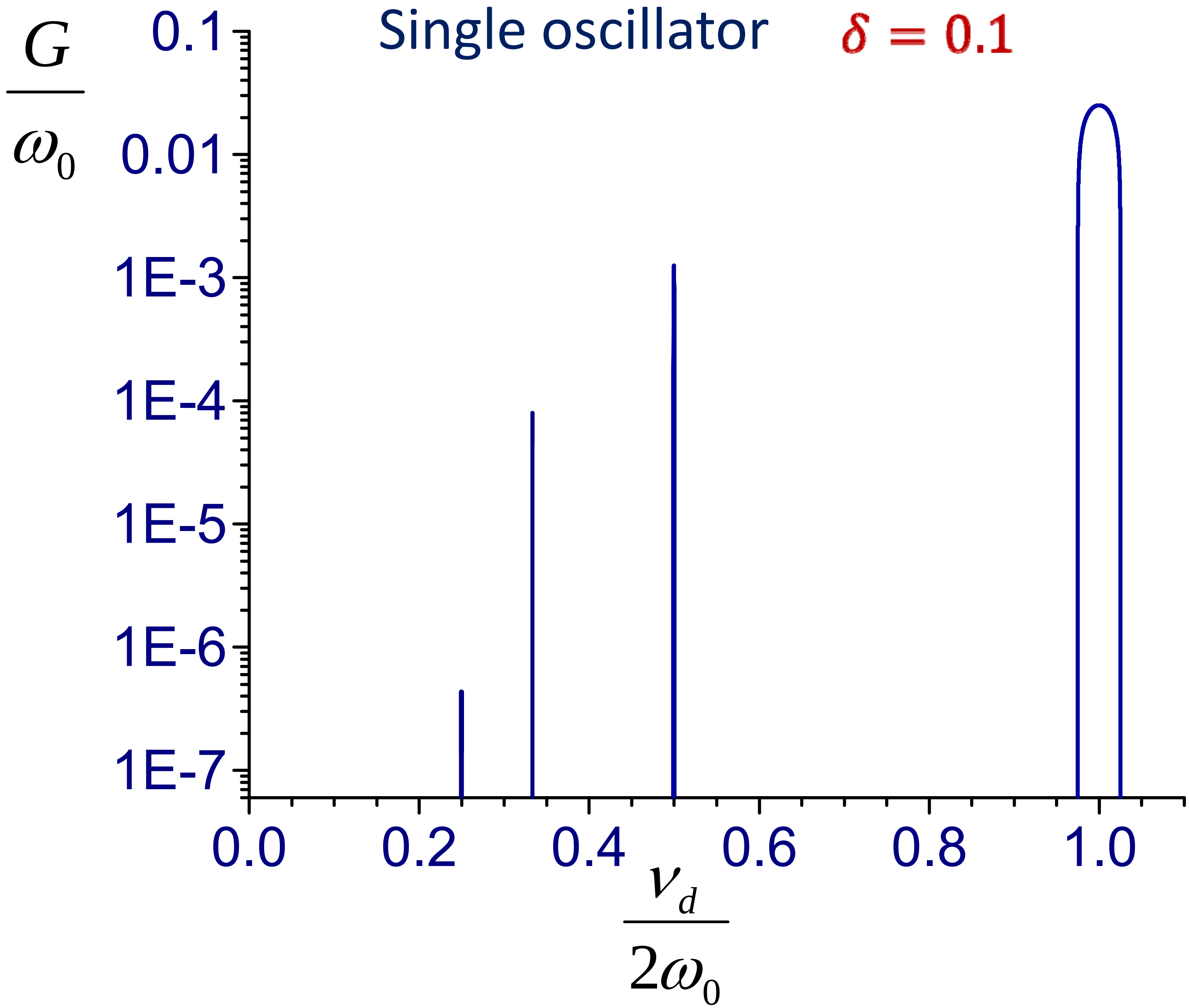
Quantum Nanosystems and Quantum Optical Probing



Mikhail Lukin, Harvard

Past: Demonstration of nanometer-scale thermometry in a living cell.
Future: Use this technique to measure magnetic fields in cell and in the brain.
Kucsko, G.; Lukin, M.D., Aggie Laser Physics Ph.D. "Nanometer-scale thermometry in a living cell", Nature 500, 54 (2013).

Gain per unit time as a function of modulation frequency



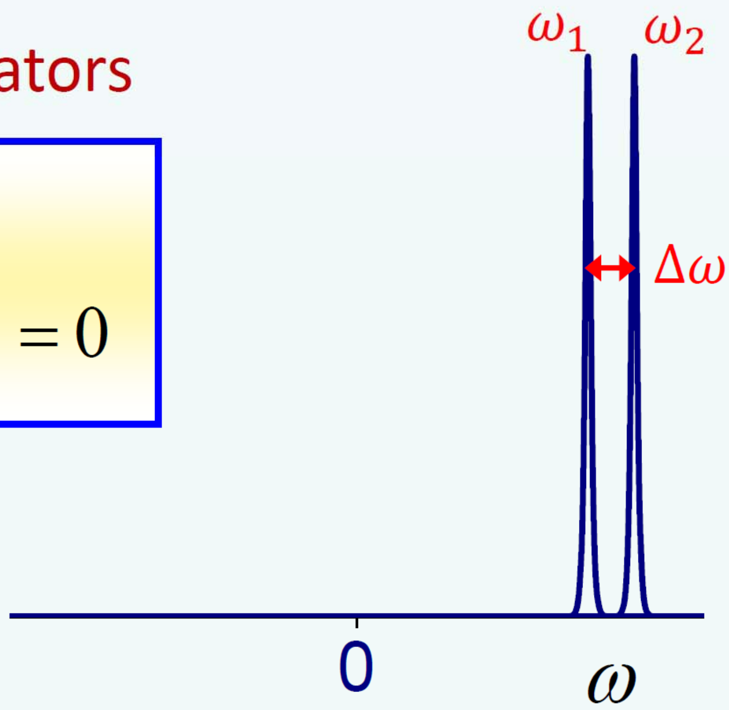
Systems with many normal modes $\omega_1, \omega_2, \omega_3, \ldots$

$\nu_d = \omega_2 + \omega_1$ ⟵ **Sum combination resonance**

$\nu_d = \omega_2 - \omega_1$ ⟵ **Difference combination resonance**

Example: Coupled parametric oscillators

$$\ddot{x}_1 + \omega_0^2 x_1 - \Omega^2 x_2 = 0$$
$$\ddot{x}_2 + \omega_0^2 x_2 - \Omega^2[1 + \delta\cos(\nu_d t)]x_1 = 0$$



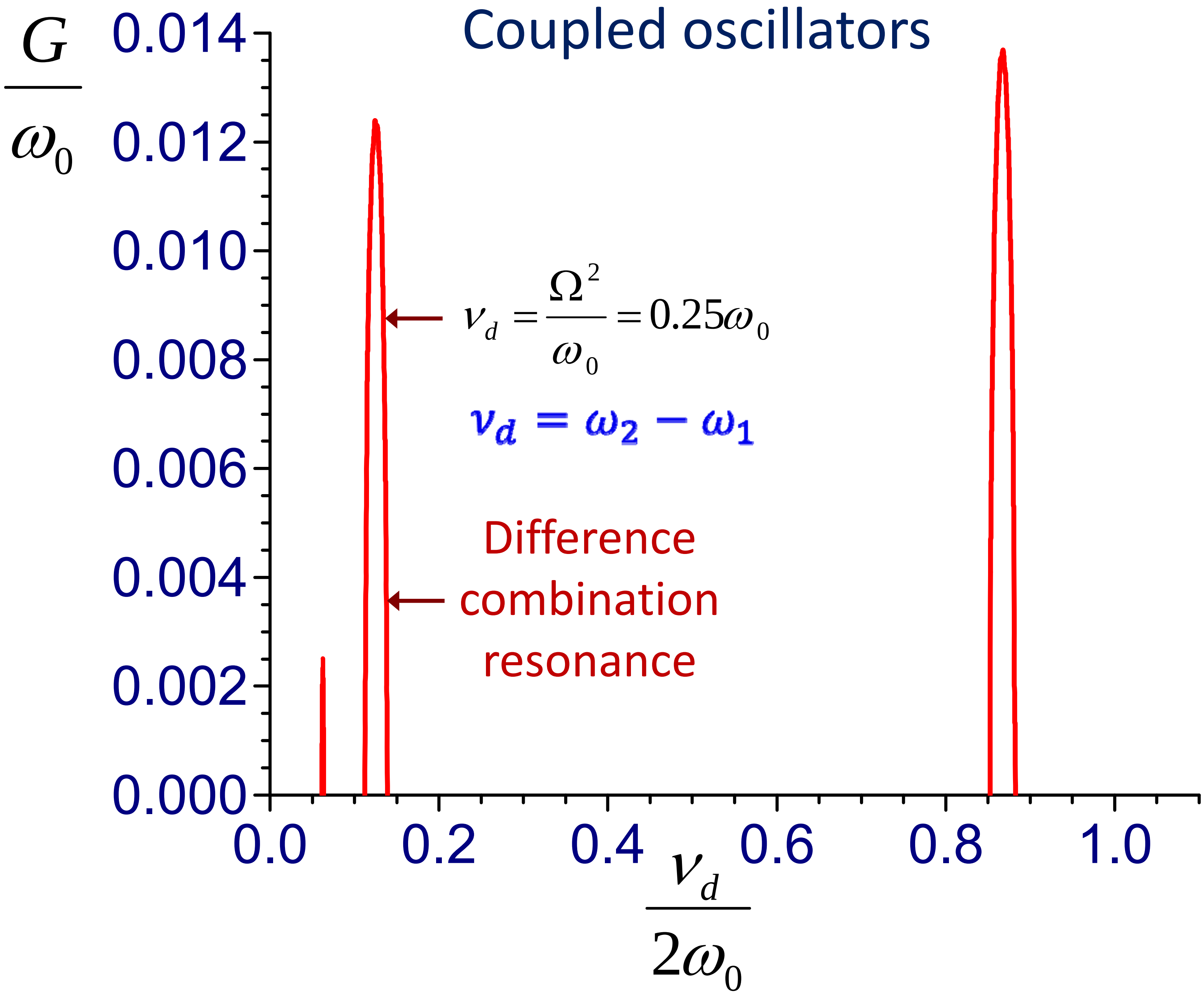
For 1st order resonance:

$$G \propto \delta \cdot \Delta\omega$$

We solve equations numerically with

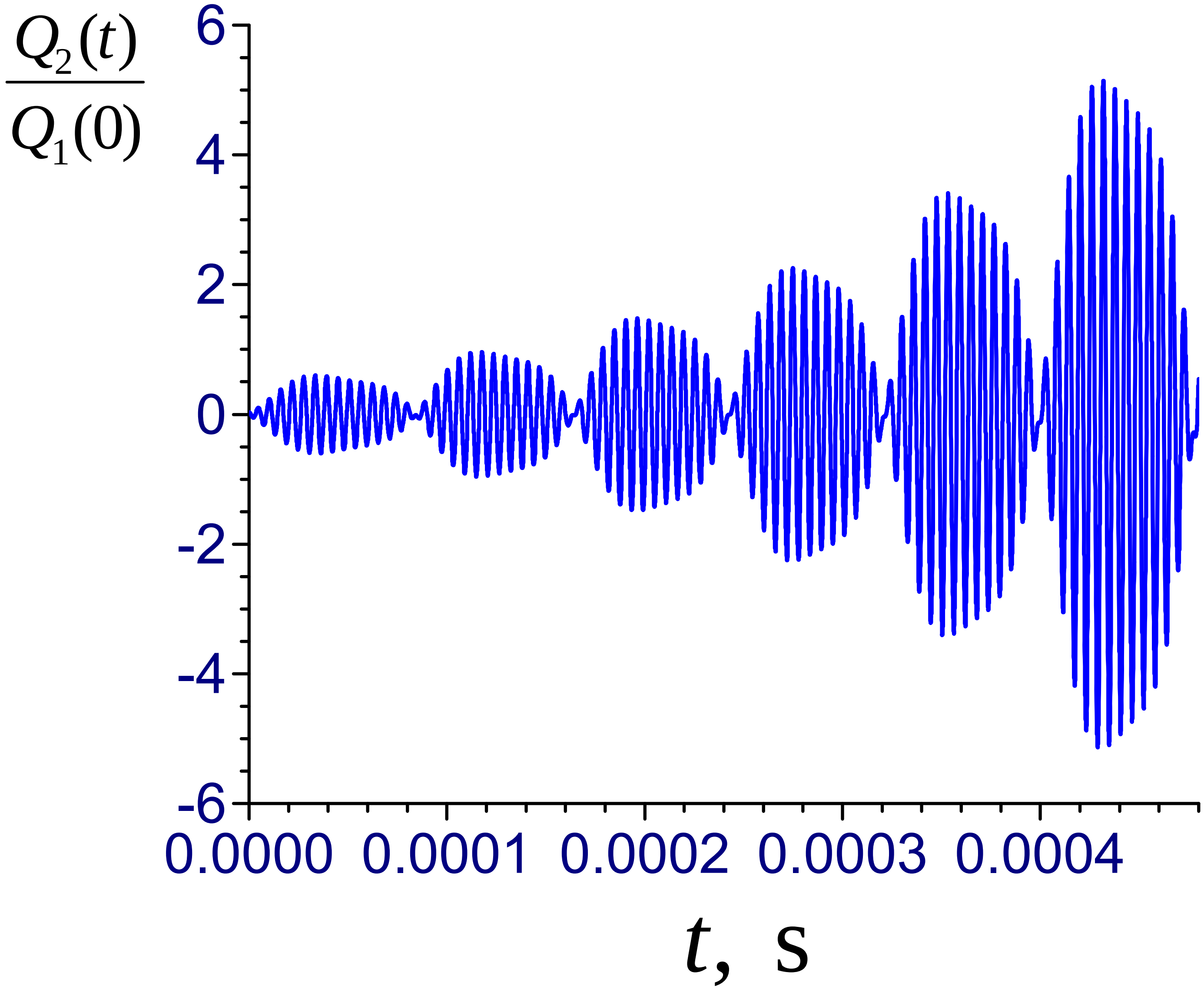
$$\frac{\Omega^2}{\omega_0^2} = 0.25, \quad \delta = 0.4$$

Gain per unit time as a function of modulation frequency

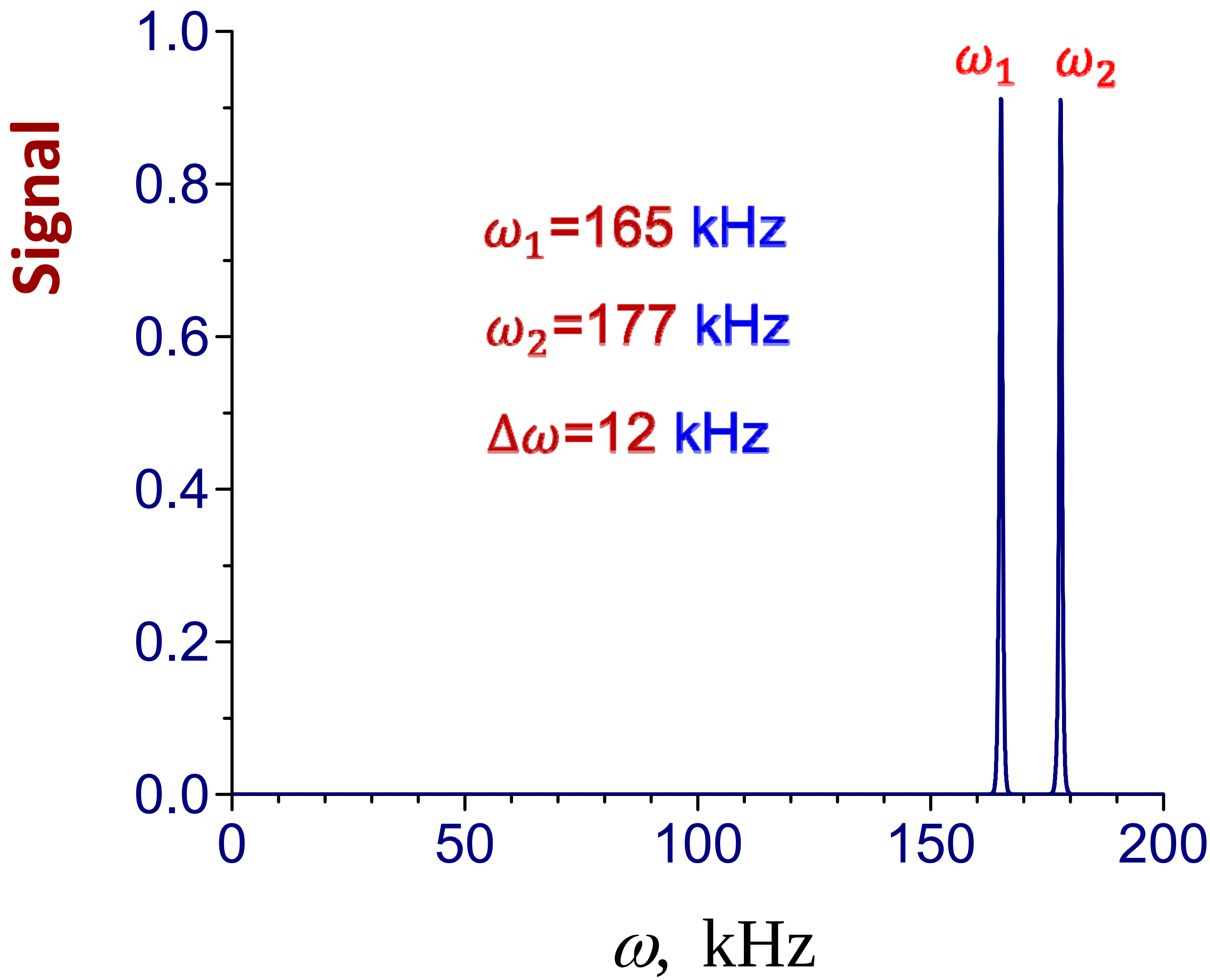


Oscillation of charge on capacitor C_2 (Theory)

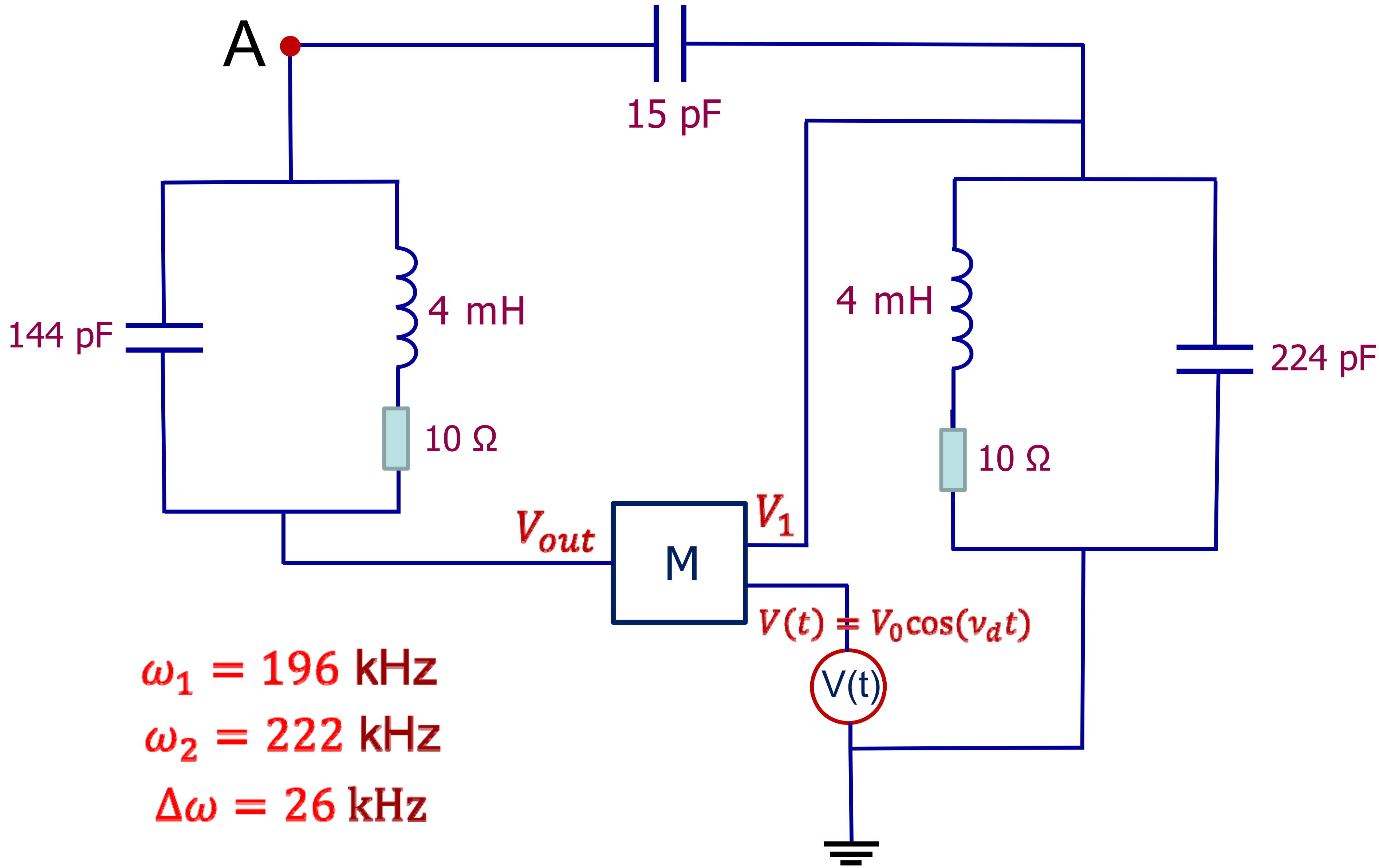
With drive $\alpha V_0 = 0.1, \nu_d = 12$ kHz



In frequency domain (Theory)



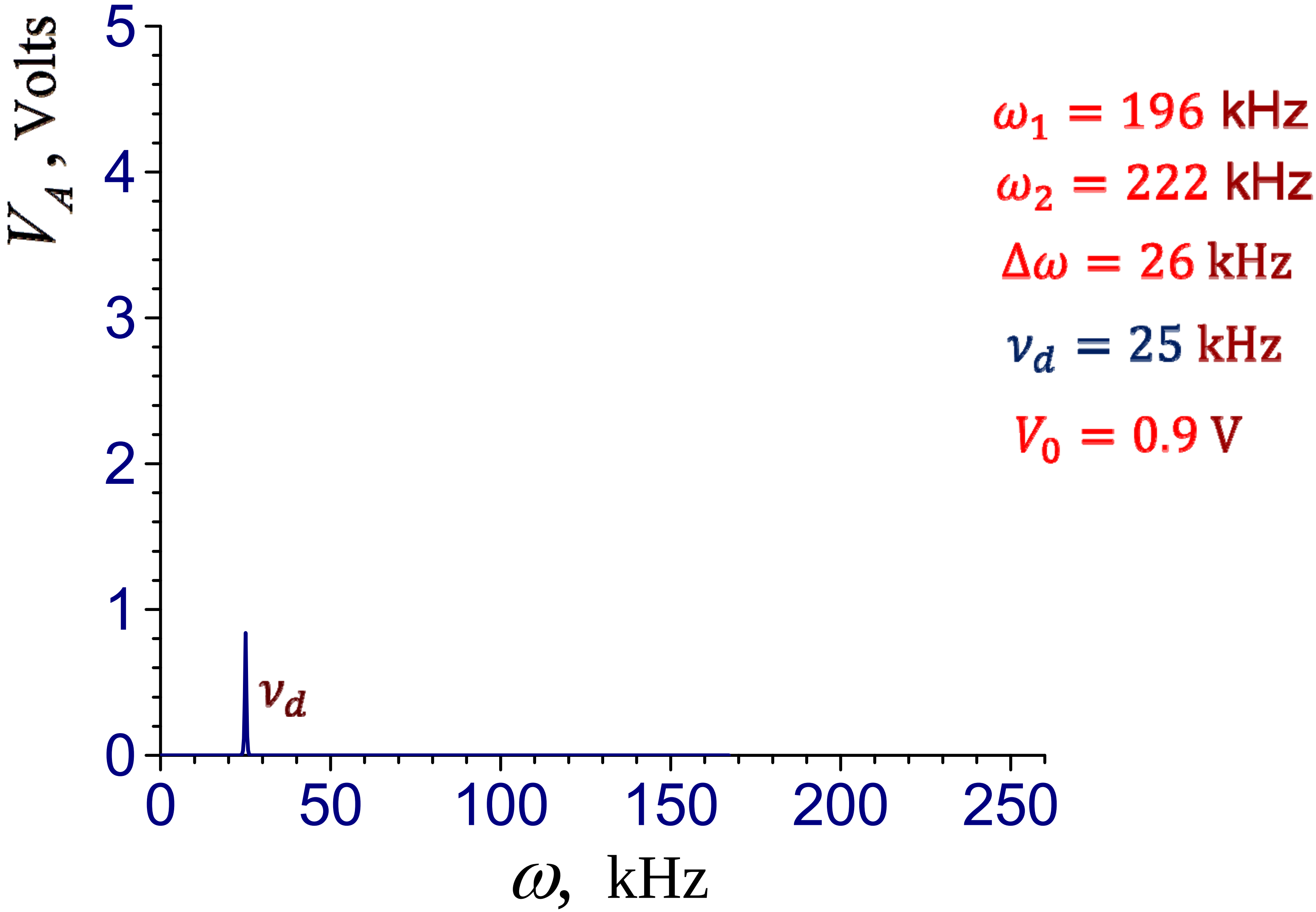
Experimental demonstration of difference combination resonance



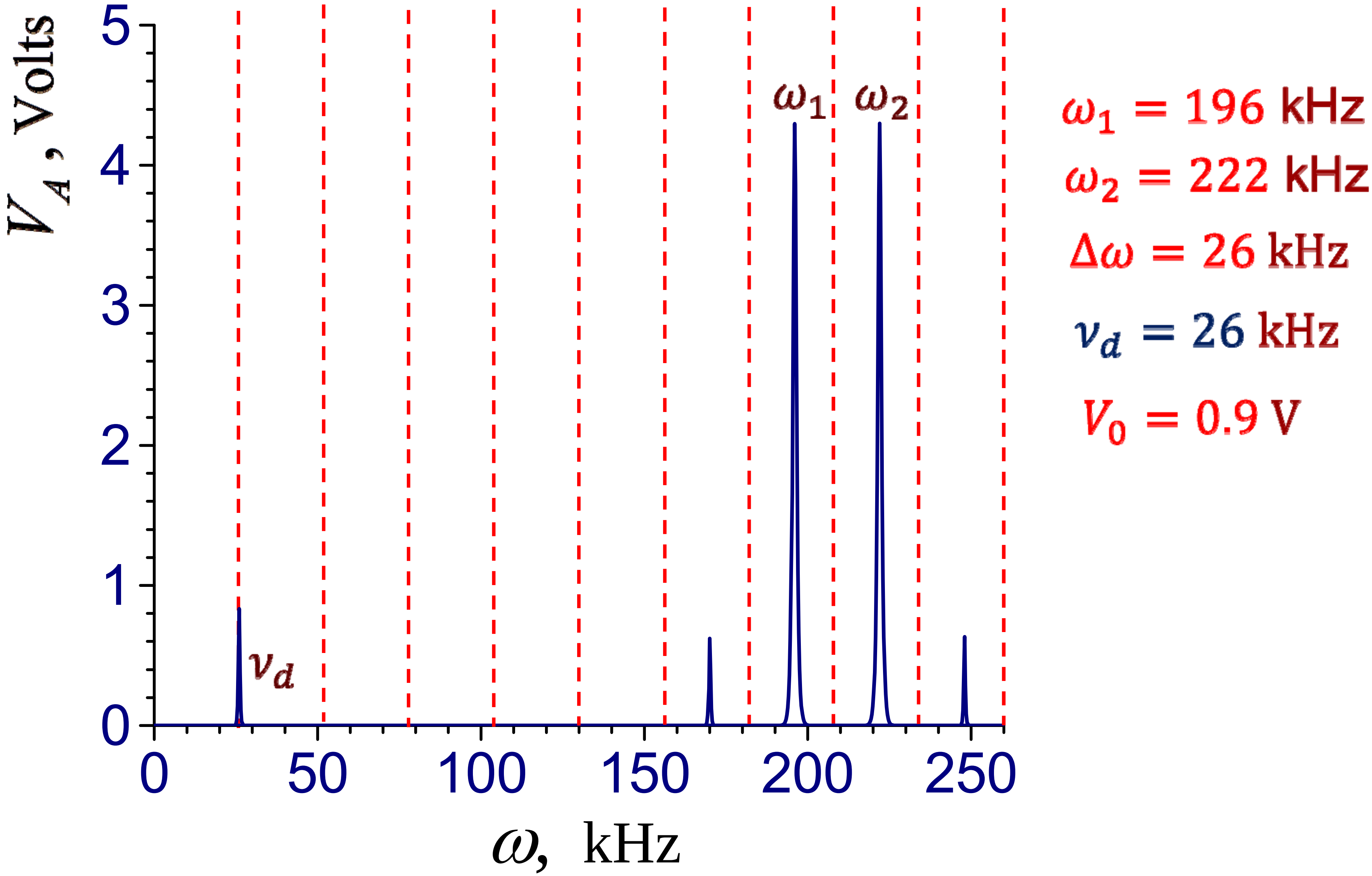
$\omega_1 = 196$ kHz

$\omega_2 = 222$ kHz

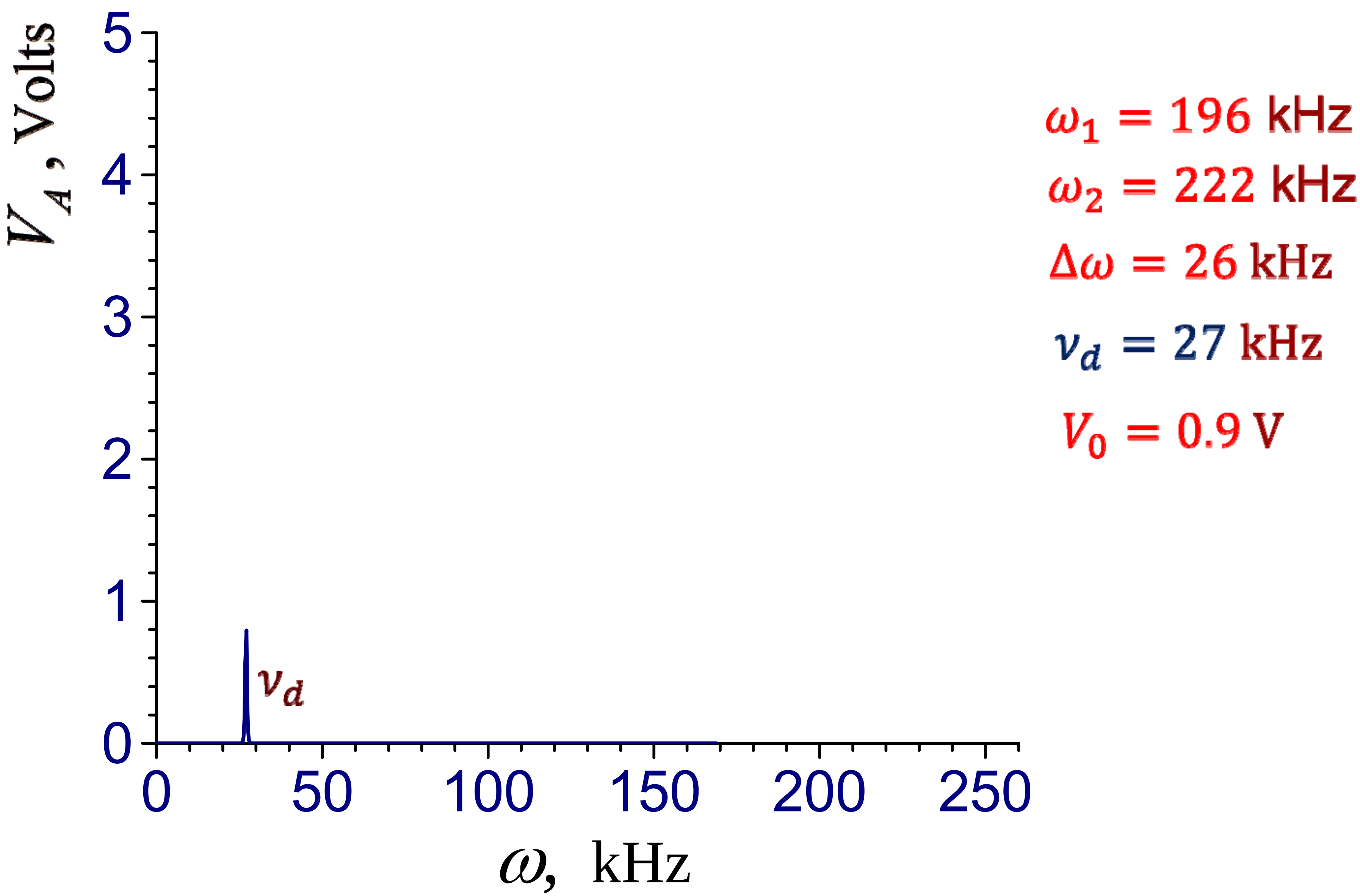
$\Delta\omega = 26$ kHz

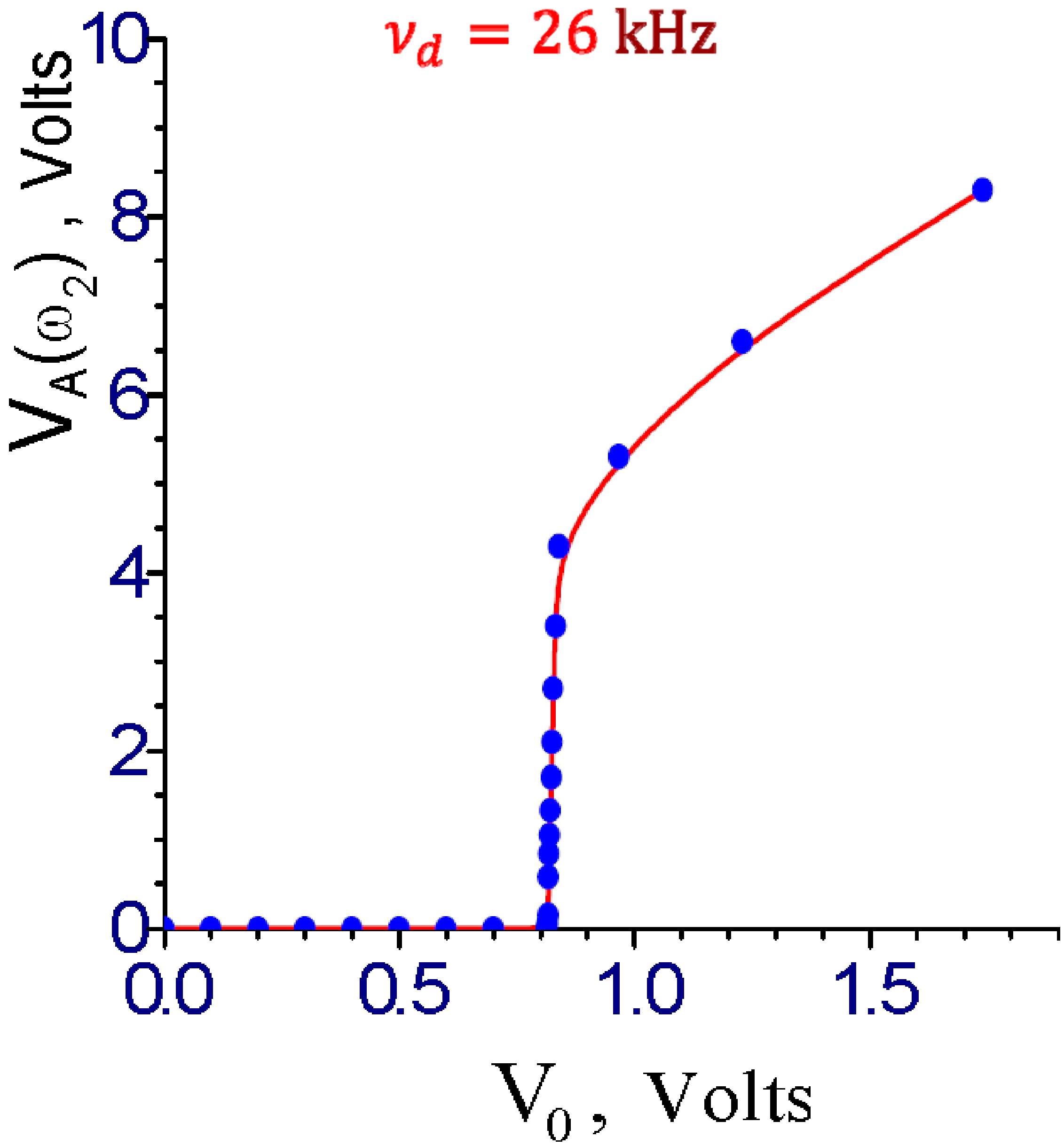


V_A , Volts
5
4
3
2
1
0
0
50
100
150
200
250
ω, kHz
ν_d
$\omega_1 = 196$ kHz
$\omega_2 = 222$ kHz
$\Delta\omega = 26$ kHz
$\nu_d = 25$ kHz
$V_0 = 0.9$ V



V_A , Volts
5
4
3
2
1
0
0
50
100
150
200
250
ω, kHz
ω_1
ω_2
ν_d
$\omega_1 = 196$ kHz
$\omega_2 = 222$ kHz
$\Delta\omega = 26$ kHz
$\nu_d = 26$ kHz
$V_0 = 0.9$ V


V_A , Volts
5
4
3
2
1
0
ν_d
0
50
100
150
200
250
ω, kHz
$\omega_1 = 196$ kHz
$\omega_2 = 222$ kHz
$\Delta\omega = 26$ kHz
$\nu_d = 27$ kHz
$V_0 = 0.9$ V



$\nu_d = 26$ kHz
$V_A(\omega_2)$, Volts
0
2
4
6
8
10
0.0
0.5
1.0
1.5
V_0 , Volts