JoVE: Science Education

Hydraulic Jumps

--Manuscript Draft--

Manuscript Number:	10405
Full Title:	Hydraulic Jumps
Article Type:	Manuscript
Section/Category:	Manuscript Submission
Corresponding Author:	Alexander Rattner UNITED STATES
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	
Corresponding Author's Secondary Institution:	
First Author:	Alexander Rattner
First Author Secondary Information:	
Order of Authors:	Alexander Rattner
Order of Authors Secondary Information:	

PI Name: Alexander S Rattner, Mahdi Nabil

Science Education Title: Hydraulic Jumps

Overview

When liquid flows along an open channel at high velocity, the flow can become unstable, and slight disturbances can cause the liquid upper surface to transition abruptly to a higher level (Fig. 1a). This sharp increase in the liquid level is called a *hydraulic jump*. The increase in the liquid level causes a reduction in the average flow velocity. As a result, potentially destructive fluid kinetic energy is dissipated as heat. Hydraulic jumps are purposely engineered into large water works, such as dam spillways, to prevent damage and reduce erosion that could be caused by fast moving streams. Hydraulic jumps also occur naturally in rivers and streams, and can be observed in household conditions, such as the radial outflow of water from a faucet onto a sink (Fig. 1b).

In this project, an open-channel flow experimental facility will be constructed. A *sluice gate* will be installed, which is a vertical gate that can be raised or lowered to control the discharge rate of water from an upstream reservoir to a downstream spillway. The flow rate required to produce hydraulic jumps at the gate outlet will be measured. These findings will be compared with theoretical values based on mass and momentum analyses.

Principles

In wide open-channel flows, liquid is only confined by a lower solid boundary and its upper surface is exposed to the atmosphere. A control-volume analysis can be performed on a section of an open channel flow to balance inlet and outlet transport of mass and momentum (Fig 2). If the velocities are assumed uniform at the inlet and outlet of the control volume (V_1 and V_2 respectively) with corresponding liquid depths H_1 and H_2, then a steady mass flow balance reduces to:

$$H_1 V_1 = H_2 V_2 \quad (1)$$

The x-direction momentum analysis of this control volume balances forces from hydrostatic pressure (due to fluid depth) with the inlet and outlet momentum flow rates (Eqn. 2). The pressure forces act inward on the two sides of the control volume, and are equal to the specific gravity of the liquid (liquid density times gravitational acceleration: ρg), multiplied by the average liquid depth on each side ($H_1/2$, $H_2/2$), multiplied the height over which the pressure acts on each side (H_1, H_2). This results in the quadratic expression on the left side of Eqn. 2. The momentum flow rates through each side (Eqn. 2, right side) are equal to the mass flow rates of liquid through the control volume (in: $-\rho H_1 V_1$, out: $\rho H_2 V_2$) multiplied by the fluid velocities (V_1, V_2).

$$\rho g \frac{(H_1^2 - H_2^2)}{2} = \rho (H_2 V_2^2 - H_1 V_1^2) \quad (2)$$

Commented [A1]: I moved things around a bit in the intro based on the requested changes. That causes Word to delete some comments, so I have reproduced them here for reference.

The original comment was: "Please open with a definition of the topic- what is a hydraulic jump and why/how is it used? What is the significance of this topic? In other words, how does this relate to Mechanical Engineering and hydraulics in general?"

The new introduction starts with a discussion of the instability, definition of hydraulic jump, effects, and why/when it is used in engineering systems. I also added a Figure to help clarify this section.

Commented [A2]: Original comment

"This statement makes it sound like a fluid rise causes the reduction of flow velocity.

But don't we want to discuss what causes the fluid rise in the first place? Isn't it really that a high flow rate stream merging with a low flow rate stream causes the fluid level to rise? And then the fluid rise reduces the average flow velocity."

Here we are thinking about hydraulic jumps that occur with just a single flow stream, not two streams that merge.

Commented [A3]: Add figure of spillway

Commented [A4]: Original comment from editor reproduced here:

"Perhaps we also add in the basic liquid pressure equation P=rho*g*h to remind the viewer where the left side comes from.

Please describe why the H terms and V terms are squared in Eqn 2.. The reader may not understand the difference between the right hand side of Eqn 2 and Eqn 1. Just a brief reminder that Eqn 1 is mass flow and Eqn 2 is momentum flow should suffice."

Eqn. 1 can be substituted into Eqn. 2 to eliminate V_2. The Froude number ($\mathrm{Fr}_1 = V_1/\sqrt{gH_1}$) can also be substituted in, which represents the relative strength of inflow fluid momentum to hydrostatic forces. The resulting expression can be stated as:

$$\left(\frac{H_2}{H_1}\right)^2 - \left(\frac{H_2}{H_1}\right)(2\mathrm{Fr}_1^2 + 1) + 2\mathrm{Fr}_1^2 = 0 \quad (3)$$

This cubic equation has three solutions. One is $H_1 = H_2$, which gives the normal open-channel behavior (inlet depth = outlet depth). The other solutions correspond to increases in depth (hydraulic jump) and decreases in depth (hydraulic depression). If the inlet Froude number (Fr_1) is greater than one, the flow is called supercritical (unstable) and has high mechanical energy (kinetic + gravitational potential energy). In this case, a hydraulic jump can form spontaneously or due to some disturbance to the flow. The hydraulic jump dissipates mechanical energy into heat, significantly reducing the kinetic energy and slightly increasing the potential energy of the flow. The resulting outlet height is given by Eqn. 4 (a solution to Eqn. 3). A hydraulic depression cannot occur if $\mathrm{Fr}_1 > 1$ because it would increase mechanical energy of the flow, violating the second law of thermodynamics.

$$H_2 = H_1 \frac{\left(\sqrt{1+8\mathrm{Fr}_1^2}-1\right)}{2} \quad (4)$$

The strength of hydraulic jumps increases with inlet Froude numbers. As Fr_1 increases, the magnitude of H_2/H_1 increases and a greater portion of inlet kinetic energy is dissipated as heat [1].

Procedure

Note: This experiment uses a relatively powerful submersible pump. The pump should only be plugged into a GFCI outlet to minimize electrical risks. Ensure that no other A/C powered devices are operating near the experiment.

1. Fabrication of open-channel flow facility and tank (see diagram and photograph, Fig. 3)

   1.1. Cut lengths of ~6.0 mm thick × 9.5 cm wide clear acrylic sheet with the following lengths: 2×15 cm, 2×25 cm, 1×34 cm, 1×41 cm (Fig. 3a). It is recommended to use a table saw or laser cutter to ensure that the edges are relatively flat and the sheets have equal thickness.

   1.2. Cut holes in the lower right corners of the two 60 × 45 cm acrylic sheets to mount the flow meter (Fig. 3a). Cut a hole on the upper right side of the front sheet to install the flow control valve.

   1.3. Use acrylic cement (*e.g.*, SCIGRIP 16) to bond the acrylic panels as indicated in Fig. 3a. Ensure adequate ventilation and wear gloves when handling the acrylic cement. It is helpful to apply cement with a needle syringe and use masking tape to position panels during curing. Allow the cement to cure for 24 – 48 hours.

Commented [A5]: Original comment:

"Can you describe what the difference is between undulating, oscillating, steady and strong jumps are? Physically, what would those look like and what properties would they have- aside from the difference in Fr ranges."

Originally, I had a list of different classifications of hydraulic jumps that occur at different Froude numbers. I don't think it really added much since we don't focus on that in the analysis. I shortened this section.

Commented [AM6]: This section would be helpful for someone trying to replicate this experiment, but I don't think the construction of the tank should be included in the video protocol. We should simply have the viewer refer to the text for specifications on how to assemble the tank.

Commented [A7]: This sounds reasonable to me. Maybe we can just include a views of the facility from a few angles to show the important features

1.4. Install the flow meter on the front panel and affix with the provided screws. Install 1 NPT to ½ NPT reducing fittings on the flow meter inlet and outlet ports. Install ½ NPT to 0.5 in. inner diameter barbed fitting adapters to those fittings.

1.5. Install a 0.5 in. ID and a 0.75 in. ID barbed fitting onto the gate valve (flow rate control). Connect the barbed fitting to the submersible pump with a ~20 cm length of tubing so that the valve handle lines up with the hole on the top right of the acrylic enclosure (Fig. 3b-c).

1.6. Insert the pump into the lower reservoir, and install the valve so that the valve stem passes through the mounting hole and the handle is outside the enclosure (Fig. 3c).

1.7. Insert a vertical acrylic panel near the inlet portion of the flow facility so that there is approximately a 5.0 mm opening below it (Fig. 3b-c). This component will act as the *sluice gate*, and can be raised and lowered to control flow from the upper reservoir to the channel.

1.8. Fill the upper reservoir loosely with a stainless steel wool scouring pad. This helps distribute inlet water flow evenly across the channel.

1.9. Connect the valve outlet to the flow meter inlet with a length of soft plastic tubing. Connect the flow meter outlet to the upper reservoir with plastic tubing. Ensure that the tubing inlet to the upper reservoir is well anchored so that it does not swing out when the pump is turned on.

1.10. Fill the lower reservoir with water.

2. Performing experiment

2.1. Measure the gap height underneath the gate using a ruler, and denote the value as H_1.

2.2. Turn on the pump, and adjust the flow rate using the valve to various flow rates (5 – 15 l min^{-1}). Use a ruler to measure the liquid depth downstream from the gate (H_2) for each case.

2.3. Qualitatively observe the shapes of the hydraulic jumps that form at different flow rates. Watch for the minimum threshold flow rate for formation of a hydraulic jump. Sharper, greater amplitude ($H_2 – H_1$), jumps should occur at higher flow rates.

3. Data Analysis

Commented [A8]: Clarify sluice gate in figure

Commented [AM9]: I would start the video protocol at step 1.4.

Commented [A10R9]: This sounds reasonable to me.

Commented [AM11]: Isn't this measured quantitatively as H2?

Commented [A12R11]: Here I was thinking more of a qualitative observation of the shape to go along with the quantitative depth measured in the previous step. This goes a bit with the different types of jumps described at the end of the principles section.

I changed the wording a bit to clarify.

3.1. For each flow rate case, calculate the inlet velocity, V_1, from the volumetric flow rate. $V_1 = \dot{V}/(H_1 W)$ where $\dot{V}$ is the volume flow rate and W is the channel width.

3.2. Evaluate the inlet Froude number ($\mathrm{Fr}_1 = V_1/\sqrt{gH_1}$) and theoretical downstream liquid depth for each case (Eqn. 4). Compare these values with measured downstream jump depths.

Commented [A13]: Comment: "Are you saying that Fr should be ~1 for all of these cases?"

Originally, I had a step: compare the measured minimum Froude number for a jump to form with the theoretical value of 1.

There is a minimum flow rate/Froude number below which jumps can't occur. The theoretical value is 1. In the experiment, a bunch of different Froude numbers may be evaluated (e.g., 0.5 – 3.0). Here, I was suggesting that the experimenter could compare the minimum Froude number for which jumps were observed with the theoretical value of 1.

I think it is better to cut this step, but talk about the behavior at different Fr in the results section

Representative Results

Upstream Froude numbers (Fr_1) and measured and theoretical downstream depths are summarized in Table 1. The measured threshold inlet flow rate for formation of a hydraulic jump corresponds to $\mathrm{Fr}_1 = 0.9 \pm 0.3$, which matches the theoretical value of 1. At supercritical flow rates ($\mathrm{Fr}_1 > 1$) predicted downstream depths match theoretical values (Eqn. 4) within experimental uncertainty.

Table 1 – Measured upstream Froude numbers (Fr_1) and downstream liquid depths for $H_1 = 5 \pm 1$ mm

Liquid Flow Rate ($\dot{V}$, l min^{-1})	**Upstream Froude Number (Fr_1)**	**Measured Downstream Depth (H_2)**	**Predicted Downstream Depth (H_2)**	**Notes**
6.0 ± 0.5	0.9 ± 0.3	5 ± 1	5 ± 1	Threshold Froude number for hydraulic jump
11.0 ± 0.5	1.7 ± 0.5	11 ± 1	10 ± 2	
12.0 ± 0.5	1.9 ± 0.6	12 ± 1	11 ± 2	
13.5 ± 0.5	2.1 ± 0.6	14 ± 1	13 ± 2	

Photographs of the hydraulic jumps from the above cases are presented in Fig. 4. No jump is observed for $\dot{V} = 6.0$ l min^{-1} ($\mathrm{Fr}_1 = 0.9$). Jumps are observed for the two other cases with $\mathrm{Fr}_1 > 1$. A stronger, higher amplitude, jump is observed at the higher flow rate supercritical case.

Summary

This experiment demonstrated the phenomena of hydraulic jumps that form at supercritical conditions (Fr > 1) in open channel flows. An experimental facility was constructed to observe hydraulic jump phenomena at varying flow rates. Downstream liquid depths were measured and matched with theoretical predictions.

In this experiment, the maximum reported inlet Froude number was 2.1. The pump was rated to deliver significantly higher flow rates, but resistance in the flow meter limited measurable flow rates to ~14 l min^{-1}. In future experiments, a pump with a greater head rating or a lower pressure drop flow meter may enable a broader range of studied conditions.

Applications

Hydraulic jumps are often engineered into hydraulic systems to dissipate fluid mechanical energy into heat. This reduces the potential for damage by high velocity liquid jetting from spillways. At high channel flow velocities, sediment can be lifted up from streambeds and fluidized. By reducing flow velocities, hydraulic jumps also reduce the potential for erosion and scouring around pilings. In water treatment plants, hydraulic jumps are sometimes used to induce mixing and aerate flow. The mixing performance and gas entrainment from hydraulic jumps can be observed qualitatively in this experiment.

For all of these applications, momentum analyses across hydraulic jumps, as discussed here, are key tools for predicting hydraulic system behavior. Similarly, scale model experiments such as those demonstrated in this project, can guide the design of open-channel flow geometries and hydraulic equipment for large-scale engineering applications.

Legend

Figure 1: **a**. Hydraulic jump occurring downstream from a spillway due to a slight perturbation to an unstable high-velocity flow. **b**. Example of hydraulic jump in radial outflow of water from a household faucet.

Figure 2: Control volume of a section of an open-channel flow containing a hydraulic jump. Inlet and out mass and momentum flow rates per unit width are indicated. Hydrostatic forces per unit width indicated in lower diagram.

Figure 3: **a**. Schematic and dimensions of facility structure. **b**. Flow diagram of hydraulic jump facility. **c**. Labeled photograph of experimental facility.

Figure 4: Photograph of hydraulic jumps, showing critical condition (no jump, $Fr_1 = 0.9$) and jumps at $Fr_1 = 1.9, 2.1$.

Materials List

Name	**Company**	**Catalog Number**	**Comments**
Equipment			
Clear acrylic sheet	McMaster	8560K226	For facility structure. 24” × 48”, 7/32” thick
Soft plastic tubing (~10 ft)	McMaster	5233K66	For majority of plumbing connections, 0.5” inner diameter.
Soft plastic tubing (~1 ft)	McMaster	5233K71	For connection pump to gate valve, 0.75” inner diameter
Gate valve	McMaster	97865K31	Flow control valve, ½ NPT connections
Barbed tubing fitting, 0.75” ID to ½” NPT	McMaster	5372K132	Connecting valve to line from pump
Barbed tubing fitting, 0.50” ID to ½” NPT	McMaster	5372K125	Connecting valve to line to flow meter, and flow meter to plastic tubing
2× Pipe reducer: 1 NPT to ½ NPT	McMaster	44705K487	Reducing fittings for flow meter

Flow meter	Uxcell	B00E0E0AS4	2-20 gal min^{-1} flow meter
High flow submersible pump	Hyperflow	Rio 20HF	
Stainless steel scouring pad	McMaster	7361T13	Flow distributor in upper reservoir
Chemicals			
SCIGRIP 16 acrylic cement	SCIGRIP	10314	Bonding acrylic panels

References

[1] J.M. Cimbala, Y.A. Cengel, Fluid Mechanics Fundamentals and Applications, 3rd edition, McGraw-Hill, New York, NY, 2014.

Figure 1

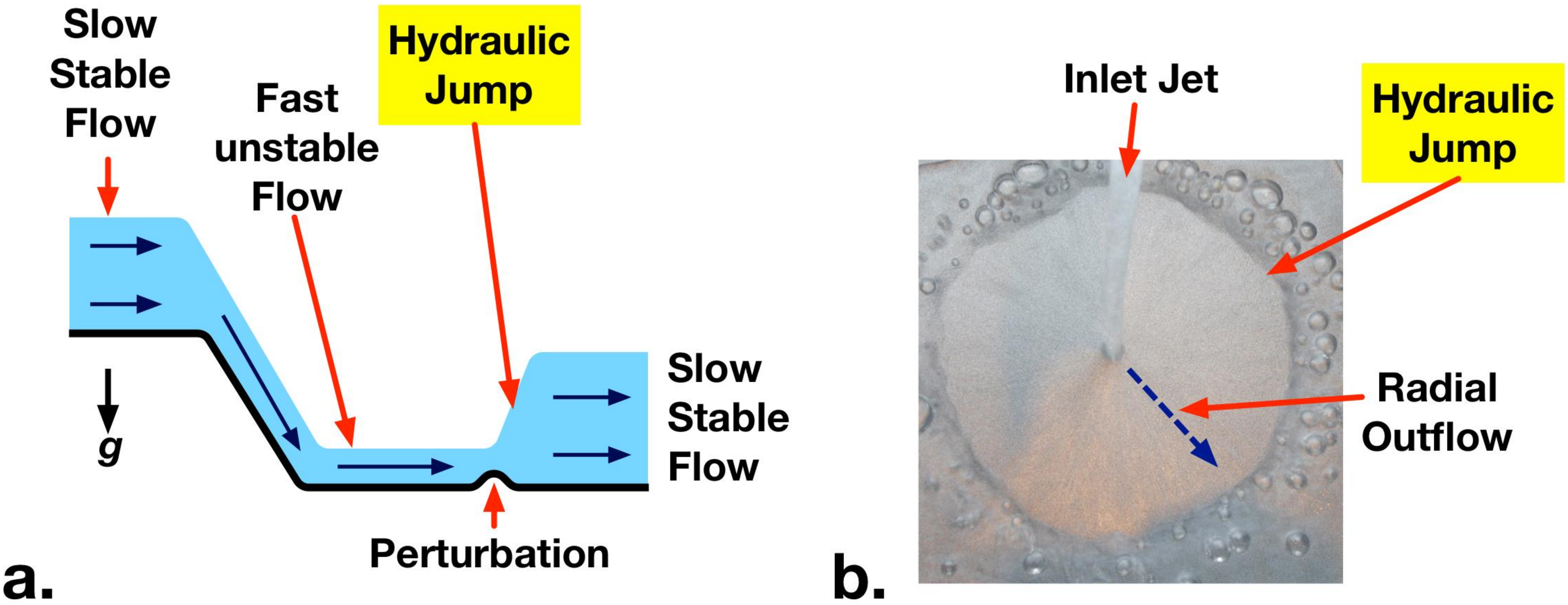
Slow Stable Flow
Fast unstable Flow
Hydraulic Jump
g
Slow Stable Flow
Perturbation
a.
Inlet Jet
Hydraulic Jump
Radial Outflow
b.

Figure 2

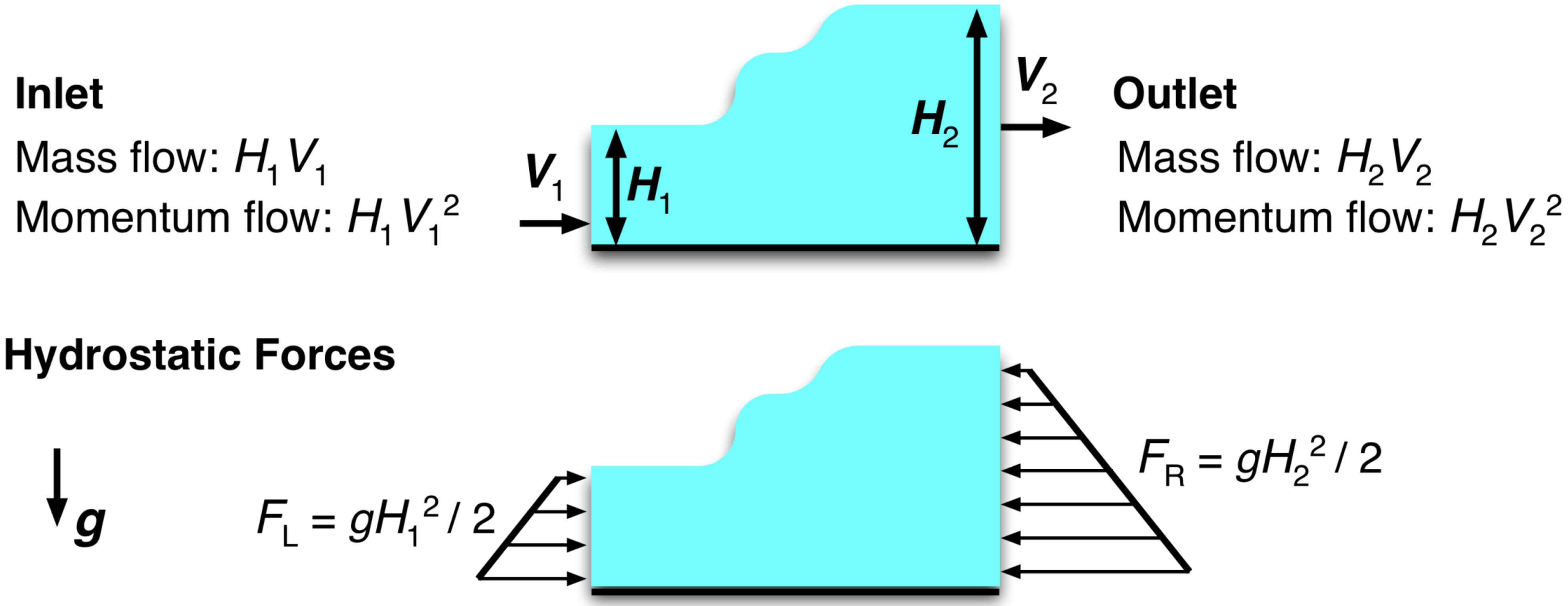
Inlet
Mass flow: $H_1 V_1$
Momentum flow: $H_1 V_1^2$
V_1
H_1
H_2
V_2
Outlet
Mass flow: $H_2 V_2$
Momentum flow: $H_2 V_2^2$
Hydrostatic Forces
g
$F_L = gH_1^2 / 2$
$F_R = gH_2^2 / 2$

Figure 3

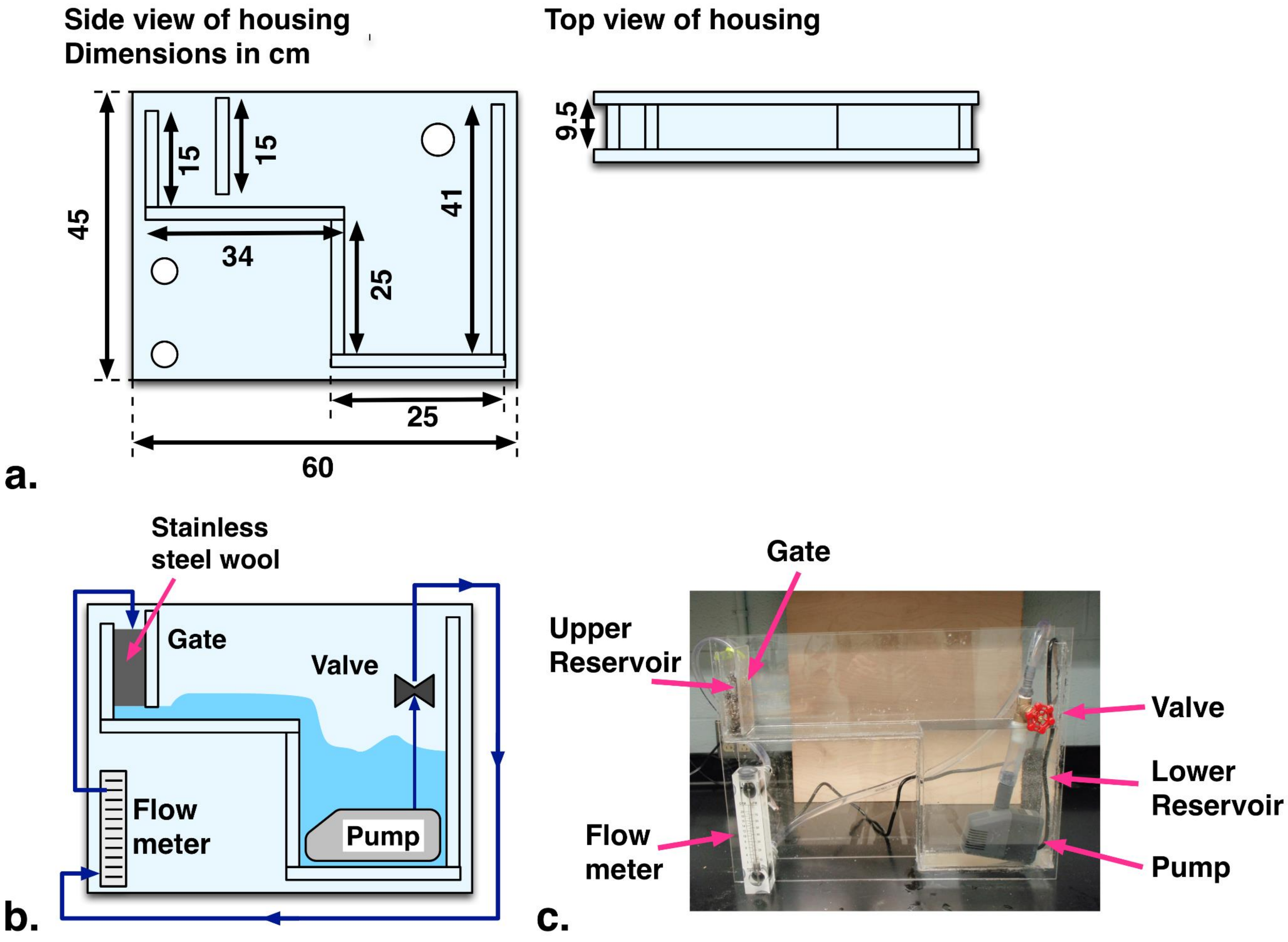
Side view of housing
Dimensions in cm
Top view of housing
45
15
15
41
34
25
25
60
9.5
a.
Stainless
steel wool
Gate
Valve
Flow
meter
Pump
b.
Gate
Upper
Reservoir
Valve
Lower
Reservoir
Flow
meter
Pump
c.

Figure 4

6.0 l min^{-1} **Fr_1 = 0.9**

12.0 l min^{-1} **Fr_1 = 1.9**

13.5 l min^{-1} **Fr_1 = 2.1**