JoVE: Science Education

Potential Energy; Springs/Oscillations

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Corresponding Author:	Asantha Cooray UNITED STATES
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	
Corresponding Author's Secondary Institution:	
First Author:	Asantha Cooray
First Author Secondary Information:	
Order of Authors:	Asantha Cooray
Order of Authors Secondary Information:	

PI Name: Asantha Cooray

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Overview:

This lab will demonstrate the potential energy stored in springs. It will also verify the restoring force equation of springs, or Hooke's Law. Potential energy is an important concept in physics. Potential energy is the energy associated with forces that depend on the position of an object relative to its surroundings. Gravitational potential energy, which is discussed in another labvideo, is the energy associated with an object that is directly proportional to the object's height above the ground. In a similar way, one can define spring potential energy, which is directly proportional to the displacement of a spring from its relaxed state. A stretched or compressed spring has potential energy, as it has the ability to do work on an object. The "ability to do work" is often quoted as the fundamental definition of energy.

The spring constant is different for springs of different elasticity. Hooke's law will be verified and the spring constant measured, by attaching varying weights to a suspended spring, and measuring the resulting displacements.
This video will demonstrate the potential energy stored in springs. It will also verify the restoring force equation of springs, or Hooke's Law. The spring constant is different for springs of different elasticity. Hooke's law will be verified and the spring constant measured, by attaching varying weights to a suspended spring, and measuring the resulting displacements.

Principles:

Commented [Amy Manoc1]: Please add discussion about oscillations. The principles discusses only the stretching of the spring, but does not discuss the effect of weight on the spring, the oscillations of a spring, etc. If a weight is added to a spring, and then released, we would see oscillatory behavior. Also, what about damping of the spring, simple harmonic motion, etc??

To hold a spring in either its compressed or stretched position, requires someone or something to exert a force on the spring. This force is directly proportional to the displacement, Δy, of the spring. In turn, the spring will exert an equal and opposite force:

$$\mathbf{F} = -k\,\Delta y, \quad \textbf{(Equation 1)}$$

where k is called the "spring stiffness constant". This is often referred to as a "restoring force" because the spring exerts a force in the opposite direction of the displacement, indicated by the negative sign. **Equation 1** is known as Hooke's lLaw, and will be demonstrated in this lab.

Simple harmonic motion will occur whenever there is a restoring force that is proportional to the displacement from equilibrium, as there is in Hooke's law. From Newton's second law, $\mathbf{F} = ma$, and recognizing the acceleration a is the second derivative of displacement with respect to time, **Equation 1** can be rewritten as

$$m\,(d^2y/dt^2) = -k\,y. \quad \textbf{(Equation 2)}$$

The solution to this second-order differential is well known to be

$$y(t) = A\sin(\omega t + \varphi) \quad \textbf{(Equation 3)}$$

where A is the amplitude of oscillation, $\omega = (k/m)^{1/2}$ and the phase angle φ depends on the initial conditions of the system. Equations of the form of **Equation 3** describe what is called simple harmonic motion. The period T, the frequency *f*, and the constant ω are related by

$$\omega = 2\pi f = 2\pi/T. \quad \textbf{(Equation 4)}$$

Thus the period T is given by

$$T = 2\pi\,(m/k)^{1/2}. \quad \textbf{(Equation 5)}$$

Note how T does not depend on the amplitude A of oscillation. So if we hung a weight from a spring suspended from the vertical, the resulting period of oscillation would be proportional to the square root of the attached weight.

The work required to stretch the spring a distance *y* is W = <**F**> *y*, where <**F**> is the average force required to stretch the string. Since **F** is linear in *y*, the average is just the force at equilibrium (=0), and the force at *y:*

<F> = ½ [0 + k*y*] **(Equation 6~~2~~).**

The work done, and hence the elastic potential energy, PE, can be written as

PE = ½ k y^2 **(Equation 7~~3~~).**

The potential energy of a spring will also be measured in this lab.

Procedure:

Commented [Amy Manoc2]: Please expand the demonstration or analysis to include discussion/calculation of the oscillation of the spring, and the calculation of the effect of weight on the potential energy, distance the spring is extended and the characteristic oscillations.

1. Measure the spring constant and potential energy of a spring, and confirm the relationship between mass and oscillatory period T.~~.~~

1.1. Obtain a spring (with a known spring constant), a stand to attach the spring to, at least 5 weights of varying mass that are able to be attached to the spring, ~~and~~ a meter stick, and a stopwatch.

1.2. Secure the stand to a solid foundation and attach the spring to the stand. Make sure there is enough room below the spring for it to stretch without hitting the table or ground.

1.3. For each of the masses, calculate the force exerted on the spring by the Earth's gravitational force (**F** = *mg*). Start with the least massive weight. Record these values in **Table 1**.

1.4. Measure how high above the surface of the table the spring is while in its un-stretched position.

1.5. Attach the least massive weight to the spring and measure the displacement Δy_1 (see **Figure 1**). Record this displacement in **Table 1**.

~~1.5.~~**1.6.** Now with the weight attached, raise the weight slightly and release it, and observe the oscillatory motion. With a stopwatch, measure the period T. For a more accurate measurement, record the time for multiple periods, and divide that time by the number of periods you observed. Do this multiple times and record the average time you measured for the period T in **Table 1.**

~~1.6.~~1.7. Repeat steps **1.5**-1.6 for all of the masses, in order of increasing mass.

~~1.7.~~1.8. Calculate the potential energy of the spring for each of the different masses and record them in **Table 1**.

1.9. Plot the force **F** as a function of displacement Δy. According to **Equation 1**, this should be linear. Fit a slope to the line. This slope will correspond to the spring constant *k*. Compare the measured value to the known value of the spring.

~~1.8.~~1.10. Using the known spring constant, calculate what the period T of oscillation should be for each of the masses and report those in **Table 1**, using **Equation 5.** Note how they compare to the T that was measured with a stopwatch in step **1.6.**

~~**(Insert Figure 1)**~~

Table 1

Mass (kg)	Weight / **F** (N)	Δy (m)	PE (J)	T measured (s)	T calculated (s)
0.5	4.9	0.49	2.4	1.3	1.4
0.75	7.4	0.74	5.4	1.6	1.7
1	9.8	0.98	9.6	1.9	1.9
1.5	14.7	1.5	21.6	2.5	2.4
2	19.6	2	38.4	2.9	2.8

Representative Results:

The representative results of the experiment with a spring of constant $k = 10$ N/m is shown in **Table 1.** The plot of **F** versus the displacement Δy is plotted below in **Figure 2**. The linear function is fit with a line, and the slope of the line is equal to the spring constant within a margin of error. The linearity of the result shows the validity of Hooke's law (**Equation 1).**

Inspect **Table 1** to see how the period T of oscillation is related to the mass that is attached to the spring. This shows that the heavier mass you attach to the spring, the longer the period will be, as it is proportional to the square root of the mass **(Equation 5)**. Also note that when you attach a larger mass to the end of the spring, the spring will be stretched further, so the potential energy of the system is larger as it is a function of the squared displacement from equilibrium **(Equation 7).** It also makes sense that the period is longer for a larger mass – because the spring is displaced further from equilibrium, it will take longer to travel that larger distance.

~~**(Insert Figure 2)**~~

Commented [Amy Manoc3]: Please expand the demonstration or analysis to include discussion/calculation of the oscillation of the spring, and the calculation of the effect of weight on the potential energy, distance the spring is extended and the characteristic oscillations.

Summary:

The displacement of a spring resulting from the application of forces of varying magnitudes was measured. The validity of Hooke's law was verified by plotting the resulting displacements as a function of the force exerted on the hanging spring. Oscillatory motion was also observed, with periods proportional to the square root of the mass attached to the spring.

Applications:

The use of springs is ubiquitous in our everyday lives. The suspension of modern cars is made from springs that are damped properly. This requires knowing the spring constants. For the smoother Cadillac rides, they use springs with a lower spring constant and the ride is “mushier”. High performance cars use springs with a higher spring constant for better handling. Diving boards are also made with springs of different spring constants, depending on how much “bounce” one wants when they dive off the board. Rock climbing ropes are also slightly elastic, so if a climber falls while climbing, the rope will not only save them from hitting the ground but it will dampen the fall with its elasticity. The smaller the spring constant of climbing rope becomes, the more closely it resembles bungee jumping.

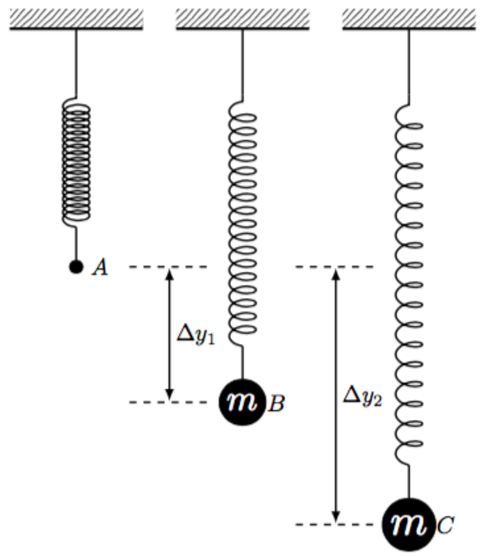

Figure 1

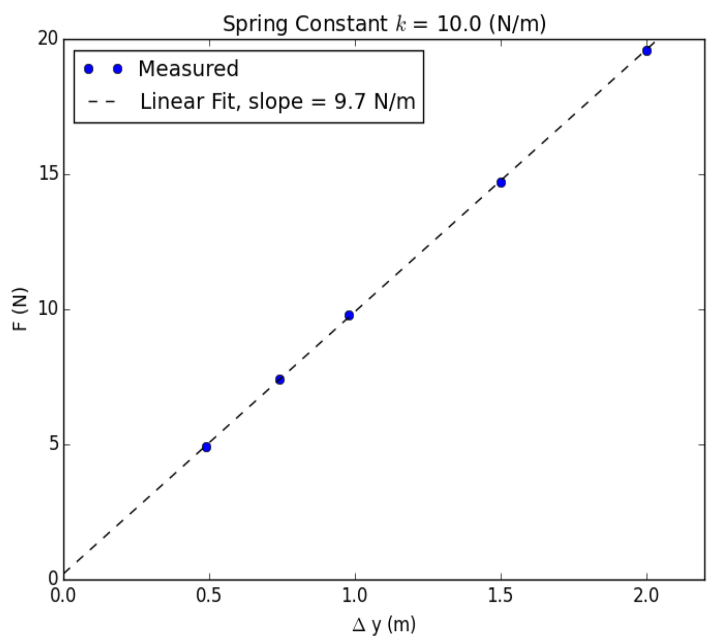

Figure 2