JoVE: Science Education

Newton's Law of Universal Gravitation

--Manuscript Draft--

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Corresponding Author:	Asantha Cooray UNITED STATES
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	
Corresponding Author's Secondary Institution:	
First Author:	Asantha Cooray
First Author Secondary Information:	
Order of Authors:	Asantha Cooray
Order of Authors Secondary Information:	

PI: Asantha Cooray

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Overview:

Legend states that Isaac Newton noticed an apple drop from a tree. He noticed the acceleration of the apple and deduced that there must be a force acting on the apple. He then surmised that if gravity can act at the top of the tree, it can also act at even larger distances. He observed the motion of the moon, then the orbits of the planets, and eventually formulated the universal law of gravitation. The law states that every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

The gravitational acceleration *g*, which is the acceleration an object on the surface of the Earth experiences due to the Earth's gravitational force, will be measured in this lab. Accurately knowing this value is extremely important, as it ~~tells us what~~describes the magnitude of the gravitational force on an object ~~is~~ at the surface of the earth.

Commented [Amy Manoc1]: Please define

Principles:

~~Legend states that Isaac Newton noticed an apple drop from a tree. He noticed the acceleration of the apple and deduced that there must be a force acting on the apple. He then surmised that if gravity can act at the top of the tree, it can also act at even larger distances. He observed the motion of the moon, then the orbits of the planets, and eventually formulated the universal law of gravitation. The law states that every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.~~
The gravitational force **F** between two masses m_1 and m_2, with their centers of mass separated by a distance **r**, can be written as

$$\mathbf{F} = G\, m_1 m_2 / r^2\ \hat{r}, \quad \textbf{(Equation 1)}$$

where $\hat{r}$ denotes that the direction of the force is pointed radially inward. Let's investigate the gravitational force between Earth and an object of mass *m* at it's surface. Using Newton's second law, **F** = *m* ***a***, the force on the mass *m* due to the Earth's gravity can be written as

$$m\,\boldsymbol{a} = G\, m\, m_E / r^2\ \hat{r}, \quad \textbf{(Equation 2)}$$

where *G* is a universal constant of proportionality that has been measured experimentally, and m_E is the mass of the Earth. In this context, the acceleration vector in that equation is typically denoted as a scalar *g*, with an implied direction pointing radially inward toward the center of the earth. For ~~us~~ people standing on the ground, ~~we typically just call~~ this direction is simply referred to as "down". Canceling the mass *m* on both sides of the equation, substituting *g* for ***a***, and noting that the distance between the object's centers of mass is just the radius of the Earth, r_E, the magnitude of the downward force can be rewritten as

$$g = G\, m_E / r^2_E. \quad \textbf{(Equation 3)}$$

Commented [Amy Manoc2]: The introduction describes the gravitational force between plants, and between a small object and the earth. Please also discuss the difference/similarities between these two cases. Which body exerts the most gravitational force on the other object?

In the famous example of the apple falling from a tree, the earth is exerting a force on the apple to make it fall, and the apple is exerting an equal and opposite force on the earth, given by **Equation 1**. The reason the Earth is essentially unaffected by the force of the apple on the Earth is because the mass of the Earth is so much larger than the apple. For larger objects, a larger force is required to make them accelerate. Hence the apple falls towards the Earth, not the Earth towards the apple. Similarly for people standing on the ground – the earth is exerting an even larger force on them as compared to the apple, and they exert an equal and opposite force on the Earth, but again because the Earth is so much more massive than a person, the gravitational force a person, or even many people, exert on the Earth essentially goes unnoticed.

This lab will demonstrate how to measure the~~is~~ acceleration g given in **Equation 3.**. Since all the quantities on the right hand side of this equation are known, the measured value of g can be compared to their product. From experiments, the values for g and G are known to be 9.8 m/s^2, and 6.67 x 10^{-11} N m^2/kg^2.

Commented [Amy Manoc3]: Please provide the values for g and G.

For this lab, a ball will be dropped and the time it takes for it to travel a known ~~measured~~ distance will be measured. From kinematics, the distance y can be written as

$$y = y_0 + v_0 t + \tfrac{1}{2} \boldsymbol{a} t^2. \quad \textbf{(Equation 4)}$$

If the ball is dropped from rest, and the acceleration $\boldsymbol{a}$ is just the gravitational acceleration, this becomes

$$y - y_0 = \tfrac{1}{2} g t^2. \quad \textbf{(Equation 5)}$$

or equivalently,

$$g = 2d / t^2, \quad \textbf{(Equation 6)}$$

where $d = y - y_0$ is the total distance traveled. G will now be experimentally determined.

Procedure:

1. Measure the acceleration of gravity at Earth's surface.

1.1 Obtain a ball, a meter stick, two timing gates, and three clamps.

1.2 Use one clamp to attach the meter stick to a table or sturdy surface, slightly off the ground.

1.3 Use the other two clamps to connect the timing gates to the top and bottom of the meter stick. Make sure that each sensor is lined up with the end of the meter stick. This way it is ~~we~~ known d = 1m in Equation 6.

1.4 Once it is verified that the timing gates are working properly, drop the ball down through the two timing gates and record the time. Make sure the ball is dropped from rest, otherwise Equation 6 is no longer valid.

1.5 Repeat step **1.4** five times and take the average time.

1.6 Use the average value of t to calculate g. Compare this to the value obtained when one uses the mass and radius of Earth in Equation 3.

Representative Results:

The measured value of g from the experimental procedure is shown in **Table 1**. The free fall time from step 1.4 is recorded in the first column of Table 1. The measured value of g is then calculated using Equation 6. The accuracy of this value can be checked by comparing it to the calculated value of g, with Equation 3, using the values for G of 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}, m_E = 5.98 x 10^{24} kg, and r_E = 6.38 x 10^3 km. This comparison is also shown in table 1 with a percent difference. The percent difference is calculated as

| measured value– expected value | / expected value (**Equation 7)**.

A low percent difference indicates that Newton's Universal Law of Gravitation is a very good description of gravity.

Table 1

Commented [Amy Manoc4]: Please provide the representative data

Free Fall Time (s)	Measured *g*	Calculated *g*	% difference
0.46	9.66	9.79	1.3

Summary:

In this experiment, the gravitational acceleration of an object on the surface of the Earth was measured. Using a ball with two timing gates attached to a meter stick, the time it takes for the ball to travel one meter from rest was measured. Using kinematic equations, the acceleration g was calculated and found to be very close to the accepted value of 9.8 m/s^2.

Applications:

The branch of mechanics that is concerned with the analysis of forces on objects that do not move is called statics. Engineers who construct building and bridges use statics to analyze the loads on the structures. The equation $\mathbf{F} = mg$ is used throughout this field, so an accurate measurement of g is extremely important in this case. Newton's law of universal gravitation is used by NASA to explore the solar system. When they send probes to Mars and beyond, they use the universal law of gravitation to calculate the spacecraft trajectories to a very high accuracy. Some scientists are interested in doing experiments in zero gravity environments. To achieve this, astronauts on the International Space Station perform the experiments for them. The space station is in a stable orbit around the Earth because of our understanding of the universal law of gravitation.