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Vectors in Multiple Directions

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Overview: This experiment demonstrates how vectors add and subtract in multiple directions. The goal will be to calculate the addition or subtraction of multiple vectors analytically and then experimentally confirm the calculations.

A vector is an object with both magnitude and direction. The magnitude of a vector is simply denoted as the length, while the direction is typically defined by the angle it makes with the x-axis. Because forces are vectors they can be used as a physical representation of vectors. By setting up a system of forces and finding which additional force will create an equilibrium between the forces a system of vectors can be experimentally verified.

Principles of Vectors: ~~**A vector is an object with both magnitude and direction. The magnitude of a vector is simply the length while the direction is typically defined by the angle it makes with the x-axis.**~~ In Figure 1, the vector $\vec{A}$ is shown, as well as the x and y--axes and the angle θ which ~~with~~ $\vec{A}$ ~~which~~ makes with the x-axis.

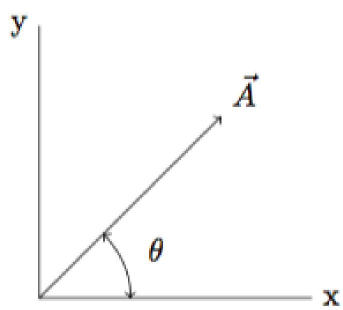

Figure 1

In order to add or subtract two vectors it is useful to describe the vector in terms of its x and y components. The x component is the amount of the vector that points in the x direction which mathematically is represented as

$$\overrightarrow{A_x} = cos(\theta) * |\vec{A}| \text{(Equation 1)}$$

The y component is represented as

$$\overrightarrow{A_y} = sin(\theta) * |\vec{A}| \text{(Equation 2)}$$

The magnitude of $\vec{A}$ is defined to be

$$|\vec{A}| = \sqrt{|\overrightarrow{A_x}|^2 + |\overrightarrow{A_y}|^2} \text{ (Equation 3)}$$

To add two vectors simply break the vector down into their x and y components and then add the corresponding components. To subtract two vectors apply the same procedure, but subtract the components.

For example: If vector $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$ and vector $\vec{B} = \overrightarrow{B_x} + \overrightarrow{B_y}$ ~~vector~~ then the addition of the two vectors $\vec{A} + \vec{B} = \vec{C} = \left(\overrightarrow{A_x} + \overrightarrow{B_x}\right) + \left(\overrightarrow{A_y} + \overrightarrow{B_y}\right)$

To determine the angle θ a vector makes with respect to the x-axis, use Equation 4.~~the formula~~

$$\overrightarrow{A_\theta} = arctan\left(\frac{|\overrightarrow{A_x}|}{|\overrightarrow{A_y}|}\right) \text{(Equation 4)}$$

Because vectors have both magnitude and direction, multiplying two vectors is not as simple as multiplying two numbers. There are two ways to multiply vectors, the dot product and the cross product. The dot product can be written as $\vec{A} \cdot \vec{B} = \left(\overrightarrow{A_x} * \overrightarrow{B_x}\right) + \left(\overrightarrow{A_y} * \overrightarrow{B_y}\right)$ or $\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \, cos(\theta)$. Here θ is the angle between the two vectors. The result only had a magnitude and not a direction. An application of the dot product in physics is work (W) where work is defined as a force times a distance $\left(W = \vec{F} \cdot \vec{D}\right)$. The cross product of two vectors can be written as $\vec{A} \times \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \, sin(\theta)\vec{n}$. While similar to the dot product the cross product contains the term $\vec{n}$ which is defined as a vector with magnitude 1 that is perpendicular to the two vectors $\vec{A}$ and $\vec{B}$. The result of the cross product is a vector. One example of the cross product in physics is torque $(\vec{\tau})$ which is the result of a force times a radius $\left(\vec{\tau} = \vec{F} \times \vec{r}\right)$.

Vectors are useful in physics because forces like gravity or friction can be represented as vectors. In this lab the force of gravity is used to demonstrate the vector nature of forces and how those forces add in multiple directions. The force of gravity on the Earth's surface is written as

$$F_{grav} = m * \vec{g} \text{(Equation 5)}$$

where m is the mass of the object while $\vec{g}$ is the acceleration of gravity near the Earth's surface $(9.8\ m/s^2)$.

Procedure:

1. Balance forces.

1.1) Set up two pulleys on the force table with the same mass facing opposite directions (180 degree difference in angle).

1.2) The force of each will be equal to $F_{grav} = m * g$. If the two forces are equal and opposite the ring at the center of the force table should not move.

1.3) Notice that if you add the components of the vectors associated with these forces you will get a resultant vector with zero magnitude. This is how you can tell all the forces are in equilibrium.

2. Analytical Calculations.

2.1) This lab will consist of three forces in equilibrium. Two forces will be known while the third will be found, first analytically using the theory of vectors, and then experimentally. For this lab keep $\vec{A}$ at zero degrees for the duration.

2.2) If $\vec{A}$ and $\vec{B}$ are known and $\vec{C}$ when added to the system causes the two forces to be in equilibrium then $\vec{C}$ is of equal magnitude **but opposite direction** of the sum $\left(\vec{A} + \vec{B}\right)$.

2.3) Calculate the magnitude of $\vec{A}$ and $\vec{B}$. Use the fact that $F_{grav} = m * g$ and that a Newton is a unit of force equal to $(kilogram) * (meter) * (second)^{-2}$.

2.4) Now using the theory of vectors calculate what magnitude $\vec{C}$ would be if it were the sum $(\vec{A}+\vec{B})$.

2.5) Now using the theory of vectors calculate what angle $\vec{C}$ would be if it were the sum $(\vec{A}+\vec{B})$.

3. Experiment.

3.1) Following the Table 1 values on the first line for $\vec{A}$ and $\vec{B}$ set up the two forces on the force table . Remember to keep $\vec{A}$ at zero degrees.

3.2) Set up the third force $\vec{C}$ by adding weights and changing the angle until an equilibrium is reached. Record these values in Table 2.

3.3) Repeat 3.2 for each of the four cases.

3.4) Determine the percent difference from the analytical result by $\frac{experimentalresult}{theoreticalprediction} \times 100$. Complete Table 2 with these calculated values.

Representative Results: The results from the lab are placed in Table 1 and Table 2.

Table 1: Analytical Results

Mass A (grams)	Magnitude $\vec{A}$ (Newtons)	Mass B (grams)	Magnitude $\vec{B}$ (Newtons)	Angle $\vec{B}$ (degrees)	Magnitude $\vec{C}$ (Newtons)	Angle $\vec{C}$ (degrees)
100	0.98	100	0.98	20	1.93	10
100	0.98	150	1.47	40	2.31	24
200	1.96	150	1.47	60	2.98	25
200	1.96	250	2.45	80	3.39	45

Table 2: Experimental Results

Magnitude $\vec{C}$ (Newtons)	Difference from analytical (%)	Angle $\vec{C}$ (degrees)	Difference from analytical (%)
2.1	9	11	10
2.2	5	26	8
2.8	6	28	12
3.5	3	43	5

The results of the experiment are in agreement with the analytical calculations. The sum of two vectors and the angle between them can be calculated using Equations 1-5. The equations are valid for making calculations of physical vectors such as force.

Summary: In this experiment the vector nature of forces is examined and measured. Vectors are added together and the resultant magnitude and direction are found both analytically as well as experimentally.

Applications: An outfielder in baseball has to understand vectors in order to catch a ball on the move. If the outfielder only knew the speed of the ball they might run to left-field instead of to right and miss the ball. If they only knew the direction of the hit they might charge in only to watch the ball sail over their head. If they understand vectors then as soon as the ball is hit they can consider both the magnitude and direction in order to estimate where the ball is going to make a catch.

When an airplane is in the sky, its speed and direction can be written as a vector. When there is a heavy wind the wind vector adds to the planes vector to give the resultant system vector. For example, if a plane is flying into the wind the magnitude of the resultant vector will be less than the plane's initial magnitude. This corresponds to the plane moving slower heading into the wind which makes intuitive sense.

When two objects collide and stick together their final momentum (a vector) can be approximated as the sum of the two initial momentum vectors. This is a simplification as in the real world two objects colliding have extra factors to consider like heat or deformation from the collision. Momentum is just the mass of an object multiplied by its velocity. If two skaters on ice traveling different directions at different speeds collide and hold onto each other their final direction and speed can be estimated based on their initial vector components.