JoVE: Science Education
Motion in 1 and 2 Dimensions
--Manuscript Draft--

Manuscript Number:	10314
Full Title:	Motion in 1 and 2 Dimensions
Article Type:	Manuscript
Section/Category:	Manuscript Submission
Corresponding Author:	Asantha Cooray UNITED STATES
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	
Corresponding Author's Secondary Institution:	
First Author:	Asantha Cooray
First Author Secondary Information:	
Order of Authors:	Asantha Cooray
Order of Authors Secondary Information:	

PI Name: Asantha Cooray

Science Education Title: Motion in 1 and 2 dimensions

Overview:

This experiment demonstrates the kinematics of motion in 1 and 2 dimensions. This lab will begin by studying motion in 1 dimension, under constant acceleration, by launching a projectile directly upwards and measuring the maximum height reached. This lab will verify that the maximum height reached is consistent with the kinematic equations derived below.

Motion in 2 dimensions will be demonstrated by launching the ball at an angle θ. Using the kinematic equations below, one can predict the distance the projectile will land based on the initial speed, total time, and angle of trajectory. This will demonstrate kinematic motion with and with out acceleration (*y*- and *x*-directions, respectively

Principles of motion in 1 and 2 dimensions:

Any measurement of an object's kinematics, such as position, displacement, and speed, must be made with respect to some reference frame. The *x*-direction of the coordinate axes will correspond to the horizontal direction, and *y* to the vertical. The origin of the coordinate axes (0, 0), will be defined as the initial position of our particle (ball).

Motion in 1 dimension

Let's begin by considering 1 dimensional motion of a ball over some particular time interval *t*, corresponding to position *y*. Denote the initial time t_0, which corresponds to position y_0. The displacement of the ball, Δy, is defined as

$$\Delta y = y - y_0 \quad \textbf{(Equation 1)}$$

The average velocity of the ball, $\bar{v}$, is the displacement divided by the elapsed time:

$$\bar{v} = (y - y_0) / (t - t_0) = \Delta x/\Delta t. \quad \textbf{(Equation 2)}$$

The instantaneous velocity, *v*, is the velocity over some very small time interval, defined as

$$v = \lim_{\Delta t \to 0} (\Delta x/\Delta t). \quad \textbf{(Equation 3)}$$

The constant acceleration, *a*, is the change in velocity divided by the elapsed time:

$$a = (v - v_0)/(t - t_0). \quad \textbf{(Equation 4)}$$

Set $t_0 = 0$ to be the initial time and solve for *v* in the last equation to obtain the velocity as a function of time as

$$v = v_0 + at. \quad \textbf{(Equation 5)}$$

Next, calculate the position *y* as a function of time using **Equation 2**. *y* is re-labelled as

$$y = y_0 + \bar{v}t. \quad \textbf{(Equation 6)}$$

Under constant acceleration, the velocity will increase at uniform rate, so the average velocity will be half-way between the initial and final velocities.

$$\bar{v} = (v_0 + v) / 2. \quad \textbf{(Equation 7)}$$

Substituting this into **Equation 6** and using the definition of instantaneous velocity, the equation for *y* becomes

$$y = y_0 + v_0 t + ½ at^2. \quad \textbf{(Equation 8)}$$

t is solved for by substituting **Equation 7** into **Equation 6**:

$$t = (v - v_0)/a. \quad \textbf{(Equation 9)}$$

Substituting that *t* into **Equation 6** and again using the definition of **Equation 7**, the equation for *y* becomes

$$y = y_0 + (v + v_0)/2\ (v - v_0)/a = y_0 + (v^2 - v_0^2)/2a. \quad \textbf{(Equation 10)}$$

Solving for v^2:

$$v^2 = v_0^2 + 2a(y - y_0). \quad \textbf{(Equation 11)}$$

These are the useful equation relating position, velocity, acceleration and time, when *a* is constant.

Motion in 2 Dimensions

Now motion in 2 dimensions will be considered. **Equations 5, 7, 8** and **11** constitute a general set of kinematic equations in the *y*-direction. These can be expanded to motion in 2 dimensions, *x* and *y*, by simply replacing the *y* components with *x* components. Consider a projectile launched with an initial velocity v_0, at an angle θ with respect to the *x*-axis, as shown in **Figure 1.** From the figure, one can see that the *x*-direction component for the initial velocity, $v_{x,0}$, is $v_0\cos(\theta)$. Similarly, in the *y*-direction, $v_{y,0} = v_0\sin(\theta)$.

the only acceleration the particle experiences is gravity, in the negative *y*-direction. So the velocity in the *x*-direction will be constant, and the velocity in the *y*-direction reaches a minimum at the peak of the parabola, half way through the displacement at *t/*2, where *t* is the total time. Use the equations above to describe this 2 dimensional motion with equations. In this coordinate frame, the origin (0,0) corresponds to (x_0, y_0). Starting with the *x*-direction,

$$x = x_0 + v_{x,0}\, t + ½\, a_x t^2. \quad \textbf{(Equation 12)}$$
$$= v_0 \cos(\theta)\, t \quad \textbf{(Equation 13)}$$

and in the *y*-direction,

$$y = y_0 + v_{y,0}\, t + ½\, a_y t^2 \quad \textbf{(Equation 14)}$$
$$= v_0\sin(\theta)t - ½\, g\, t^2 \quad \textbf{(Equation 15)}$$

(Insert Figure 1)

A projectile is launched with initial velocity v_0 at an angle θ with respect to the *x*-axis. The two velocity components are v_x and v_y, where $\mathbf{V} = v_x + v_y$.

Where *g* is the gravitational acceleration. If one measures the time it takes for the projectile to complete its path, and the angle θ and the initial velocity v_0 are known, the displacement in the *x*-and *y*-directions can be calculated. These displacement calculations will be compared to the experimental results. A similar procedure can be done in 1 dimension by shooting the projectile directly upwards, with $\theta = 0$.

Procedure:

1. Motion in 1 dimension.

1.1. Obtain a ball, a launcher with plunger, two poles, a bucket, two clamps, a bungee cord, and a two-meter stick.

1.2. Attach the launcher to a pole, with a 2-meter length of pole above it.

1.3. Use the plunger to place the ball in the launcher at maximum spring tension.

1.4. Angle the launcher directly upwards, so $\theta = 0$.

1.5. Launch the ball and use a stopwatch to measure the total time *t* it takes the ball to return to y_0.

1.6. Notice the ball reaches a maximum height of 2 meters, and stops instantaneously when it reaches that height.

1.7. Repeat steps **1.5-1.6** five times and use the average time for calculations.

2. Motion in 2 dimensions

2**.1.** Set the launcher and other pole 4 meters apart, at the same horizontal height. Attach the bucket to the other pole using the clamp and bungee cord (refer to **Figure 2**).

2.2. Use the plunger to place the ball in the launcher at maximum spring tensions.

2.3. Angle the launcher at a 45 degree angle, so $\theta = \pi/4$.

2.4. Use a stopwatch to measure the total time *t* it takes the ball to land in the bucket.

2.5. Take note of the approximate height the ball reaches.

2.6. Repeat steps **2.4-2.5** five times and use the average time for calculations.

(insert figure 2)

Representative Results

Representative results from Step 1 and 2 from the above procedure are listed below in **Table 1**. This table records the results of the maximum height the ball reached in both 1 and 2 dimensions, with a known initial velocity and total flight time. The value of the experimentally measured maximum vertical displacement is compared to that calculated using **Equation 15**, the value of which is also found below. The table also records the maximum horizontal displacement of the ball for the 2 dimensional experiment, which is compared with the calculated value from **Equation 13**, using the known initial velocity and measured flight time. It can be seen that these two results match very well, which validates the kinematic equations.

Dimensions	Measured *y* (m)	Calculated *y* (m)	Measured *x* (m)	Measured *x* (m)
1	2	2	--	--
2	1	1	4	4

Table 1: Calculated and measured results in 1 and 2 dimensions.

Summary

Kinematic equations have been derived in this experiment and their applications have been shown. The validity of the kinematic equations was verified using a ball launcher.

Applications

Kinematics is used in a wide range of applications. The military uses these kinematic equations to figure out the best way to launch ballistics. For better accuracy, the drag of air resistance is included in the equations. Car manufactures use kinematics to figure out top speed and stopping distance. Airplanes need to attain a certain speed to take off before they run out of runway. With kinematics one can compute how fast the pilot will need to accelerate when taking off at a certain airport.

Legend

Figure 1. Projectile motion in 2 dimensions.

Figure 2: Experimental setup.

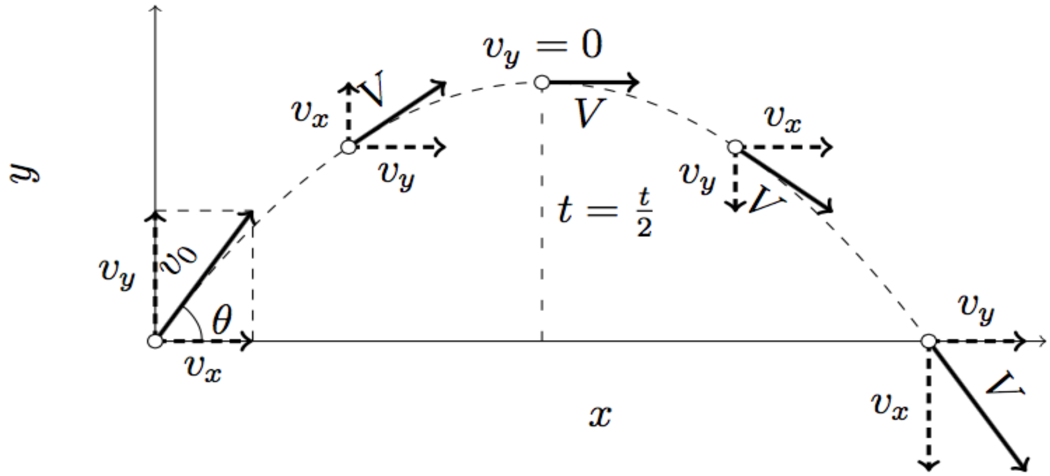

Figure 1.

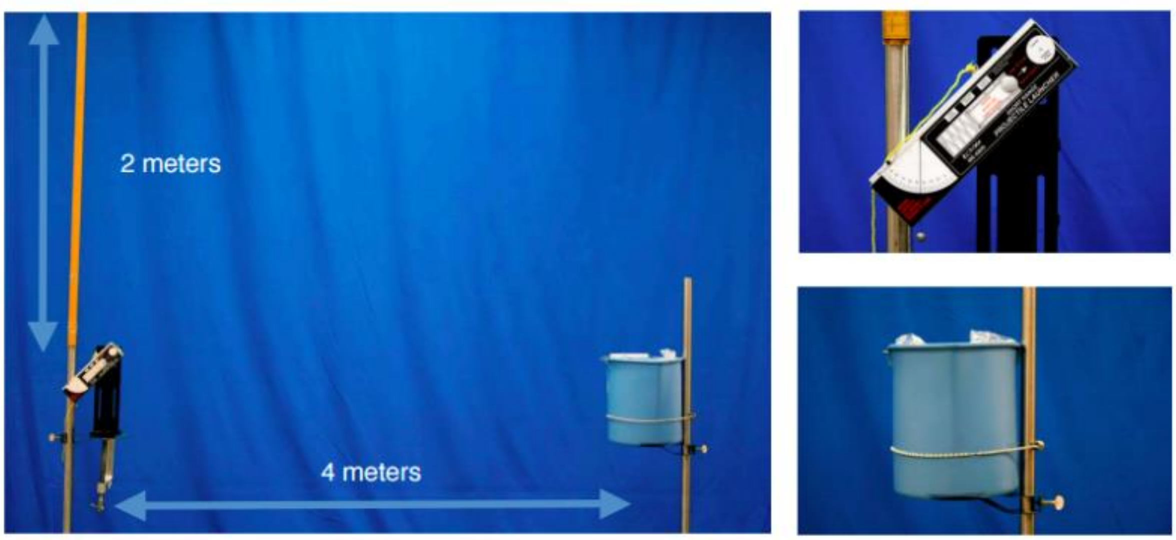

Figure 2.