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**Science Education Title**:

The Digital Oscilloscope, Part 2

**Overview**:

In physics experiments, data are often converted into voltage and recorded as functions of time. This function is not recorded continuously, however. Rather, it is sampled discretely at regular intervals by a computer and recorded digitally as a series of numbers. Handling data this way greatly simplifies the task of recording and analyzing data, but the discrete sampling necessary to do so brings its own issues. For example, if the signal varies rapidly enough, any information on how it varies between samples will be lost. Regular, rapid variations over many samples can get distorted and often appear as apparent lower-frequency signals in the digitized data record. This *aliasing* is problematic, but it is also, fortunately, well understood.

The digital oscilloscope is a simple, digital data-acquisition computer found in most physics or engineering laboratories. It can provide an almost ideal demonstration of the aliasing effect, showing quantitatively how aliasing works.

**Principles:**

In order to accurately record a continuous signal by discrete sampling, the sample points must be closely spaced compared with the typical variations of the original signal. If the sample points are too far apart, a condition known as *undersampling* occurs, and information about the original signal is lost. This situation is illustrated in Figure 1, where the original signal is shown in red and the sampled data as black dots. It is possible to draw a curve, shown in blue, through the data points that are entirely different from the original signal. If all the experimenter has knowledge of are the discrete data points, it is impossible to tell which curve (the red, high-frequency one or the lower-frequency blue) the data were sampled from. In effect, the high-frequency signal is masquerading as a lower-frequency one. The situation can be remedied either by sampling at a higher rate or by introducing a filter to remove any frequency components of the signal that are high enough to exhibit this effect. Such a filter is called an *anti-aliasing filter*, and its design requires both a knowledge of the sampling rate and an understanding of how that rate relates to measurements of signals at various frequencies.

The relationship between sampling rate and the highest-frequency signals that can accurately be recorded is described in the *Nyquist-Shannon Sampling Theorem*, which states:

Any continuous function *f(t)* is only uniquely determined by a set of discretely-sampled points if the sampling rate R is greater than twice the highest-frequency component of *f(t)*.

If the sampling rate is exactly twice the frequency of a sine wave, it can result in either a sawtooth reconstruction or a flat line, depending on the phase (peaks vs. nodes), so the *fidelity* is not good for these high-frequency signals (relative to the sampling frequency).

**Procedure:**

1. Turn on the power to both the function generator and the oscilloscope.

2. Set the horizontal scale of the scope to 2.0ms/div and the vertical scale to 500mV/div.

3. Set the function generator to output a sine wave with a frequency of 1kHz and an amplitude of 1V.

4. Using a BNC cable, connect the function generator SIG-OUT to Channel 1 of the scope.  
  
5. Observe the trace on the screen. Both frequency and amplitude should agree well with what the function generator is programmed for.   
  
6. Note that, depending on the function generator, amplitude might be specified normally (i.e. zero to max height), RMS (root-mean-squared), or peak-to-peak. In this experiment, it does not matter which is used. All are within a factor of two of each other, which works fine for this demonstration.

7. Calculate the sampling rate. The manual for the TDS3012B scope states that a trace contains 10^4 points over the whole screen. This corresponds to 10^3 points per division. The sampling rate, therefore, is 10^3 samples/div divided by 2.0ms/div, or 5\*10^5 samples per second. We may abbreviate this as 500 kilo-samples per second.

8. Calculate the Nyquist frequency for this scope setting. For accurate sampling at these settings, the highest-frequency component of the signal must be half the sampling rate, or 250 kHz.

9. Increase the frequency on the function generator until it reaches 250 kHz, observing the screen after each change. The period of the sine wave gets shorter and shorter until individual periods are no longer clearly identifiable. Aliasing begins at the Nyquist frequency.

10. Increase the frequency further. At some point, the scope will cleanly alias the signal down to a frequency that shows a long, clear period on the screen, but the triggering will be irregular.

11. Change the timebase of the scope, lowering the seconds per division, until the signal becomes resolved again.

**Results**

**Figure 2**

**Figure 3**

**Applications:**

Aliasing is not usually exploited. Rather, it is something to be avoided whenever possible. This is typically addressed in the context of a measurement with a fixed sample rate. When the sample rate is fixed (and known, of course) a special *anti-aliasing filter* is introduced just before the point where the signal is digitally sampled. This filter is most often a simple, low-pass filter that suppresses frequencies above the Nyquist frequency of the (fixed) sample rate.

The oscilloscope does not contain such a filter, because it has a variable sampling rate, and a corresponding variable-frequency anti-aliasing filter would generally prove cumbersome and expensive. While this is something the experimenter must be aware of when taking data with a scope, it also provides an excellent opportunity to demonstrate this important effect, preparing for both further electronics design work and for data analysis of digitized signals.

**Legend:**

Figure 1: Example of aliasing. The high-frequency signal (red) is "undersampled," i.e. data points are not taken at close enough intervals to accurately reconstruct the original signal, and the reconstructed signal appears at a lower frequency (blue).

Photo Credit: Image courtesy of Wikipedia.org (url: http://upload.wikimedia.org/wikipedia/commons/2/28/AliasingSines.svg).

Figure 2: Output of the scope with the function generator set at exactly the Nyquist frequency.

Figure 3: Output of the scope with the function generator set half a kilohertz below twice the Nyquist frequency, or 499.5 kHz. Note the aliased frequency appears to only be 500 Hz.